Nalanda Open University

B.SC Part-3

Course : Physics(Hons)

Paper : 8

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Topic- Thermionic emission (Electronics)

Thermionic emission

Thermionic emission must not be confused with thermic emission (of black body for example) where photons are emitted from heated body. In the case of thermionic emission we observe emitted electrons from a metal due to its high temperature.

Mobile electrons in metals, also called valence electrons, are responsible for electric current conduction. If we increase the temperature of the metal, electrons start to move faster and some may have enough energy to escape (evaporate) from the metal. The higher the temperature, the higher will be the current of escaping electrons. This temperature induced electron flow is called thermionic emission.

If we use the hot metal from which electrons evaporate as a cathode on a high potential difference to the anode, then we get electric field between cathode and anode and emission of electrons can be measured as a anode current.

Besides of showing relations of parameters in thermionic emission, the object of this seminar is also to show how to estimate charge over mass ratio of an electron and work function of tungsten with the simple experiment using phenomenon of thermionic emission. We can take advantage of both mentioned phenomena using ordinary double filament headlamp bulb with one filament burn-out.

Richardson's law

Potential barrier at the metal surface tends to prevent free electrons from escaping at low temperatures. When the metal is heated to sufficiently high temperature, some of free electrons get enough energy to carry them over potential barrier. With suitable electric field these electrons can be then drawn away from metal and measured. Electrons emitted by the filament are drawn to the anode, then the anode current is controlled primarily by the filament temperature and is practically independent of the potential. Under these conditions anode current is said to be saturated, and is purely a function of the temperature. [1]

Richardson's law tells us what is the current density J_x [A/m^2] of thermically escaped electrons in the direction perpendicular to heated metal. We can show how the law can be derived from basic properties of electrons in a metal.

Let us say we have a flow of thermal electrons with charge q in the x direction. We consider empty space in front of metal plate. Then the current density of this electrons is

$$J_x = \int q\, n(E)\, v_x(E)\, \mathrm{d}E \,, \qquad (1)$$

where $n(E)$ is the density of electrons in units of [J^{-1}m^{-3}]. $v_x(E)$ is the speed of electrons in x direction. The integral is taken over all electron energies needed for escaping over potential barrier in x direction.

From statistical physics we know that density of particles can be written as

$$n(E) = g(E) f(E), \tag{2}$$

where $g(E)$ is density of states and $f(E)$ is the probability for a taken state with energy E. In case of electrons that are half spin particles both functions have to obey Fermi-Dirac statistics and are written as

$$g(E) = \frac{8\sqrt{2}\pi}{h^3} m^{3/2} \sqrt{E}, \tag{3}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}, \tag{4}$$

where h and k are Planck's and Boltzmann's constants. E_F is Fermi energy, that is the highest energy of electrons in constant confining potential U (figure 2.1). Only electrons with the highest energies $E >> E_F$ can escape from metal, so $f(E)$ approaches Boltzmann's distribution:

$$f(E) = \exp\left(-\frac{E - E_F}{kT}\right). \tag{5}$$

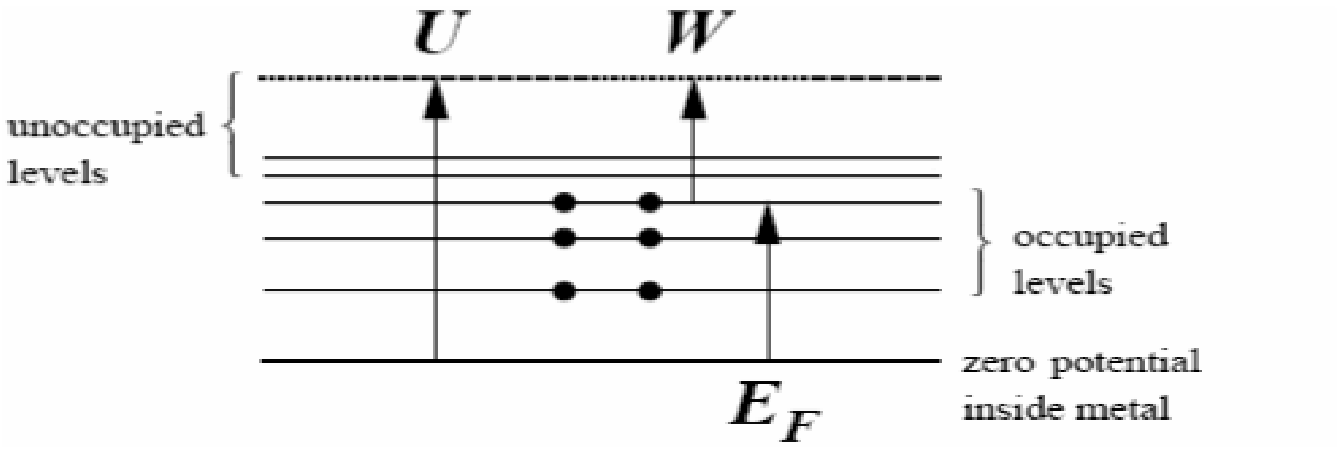

Figure 2.1: Free electrons inside metal are treated as in potential well at constant confining potential U. They fill all available states up to the Fermi energy E_F. Electrons with the highest energies lack exactly W (work function) to leave metal.

We can now write part of the equation (1) as

$$n(E)\,\mathrm{d}E = \frac{8\sqrt{2}\pi}{h^3} m^{3/2} \sqrt{E} \exp\left(-\frac{E - E_F}{kT}\right) \mathrm{d}E \,. \tag{6}$$

Let us write the energy as a function of the electron velocity

$$E = \frac{1}{2} m v^2 \Rightarrow \sqrt{E}\,\mathrm{d}E = \frac{1}{\sqrt{2}} m^{3/2} v^2 \,\mathrm{d}v \tag{7}$$

and insert it into (6):

$$n(E)\,\mathrm{d}E = \frac{8\sqrt{2}\pi}{h^3} m^{3/2} \exp\left(-\frac{E-E_F}{kT}\right)\frac{1}{\sqrt{2}} m^{3/2} v^2\,\mathrm{d}v =$$

$$= \frac{8\pi}{h^3} m^3 \exp\left(-\frac{E-E_F}{kT}\right) v^2\,\mathrm{d}v\,. \tag{8}$$

We can now insert expression (8) into equation (1) to get

$$J_x = \int q\, v_x(E) \frac{8\pi}{h^3} m^3 \exp\left(-\frac{E-E_F}{kT}\right) v^2\,\mathrm{d}v =$$

$$= \int q\, v_x(E) \frac{8\pi}{h^3} m^3 \exp\left(\frac{E_F}{kT}\right) \exp\left(-\frac{E}{kT}\right) v^2\,\mathrm{d}v =$$

$$= q \frac{2}{h^3} m^3 \exp\left(\frac{E_F}{kT}\right) \int v_x(E) \exp\left(-\frac{E}{kT}\right) 4\pi v^2\,\mathrm{d}v\,. \tag{9}$$

The integral over v_x starts from the minimum velocity needed to overcome the confining potential barrier U (illustrated on figure 2.1), because only electrons with velocity higher than $v_{x,min}$ will leave metal:

$$U = \frac{1}{2} m v_{x,\min}{}^2 \Rightarrow v_{x,\min} = \sqrt{\frac{2U}{m}}\,. \tag{10}$$

We express energy as a function of the velocity components in all three directions so that we can integrate over each velocity component. So from equation (9) on we can write

$$J_x = \frac{2qm^3}{h^3}\exp\left(\frac{E_F}{kT}\right)\int_{v_{x,\min}}^{\infty} v_x \exp\left(-\frac{mv_x^2}{2kT}\right)\mathrm{d}v_x \int_{-\infty}^{\infty}\exp\left(-\frac{mv_y^2}{2kT}\right)\mathrm{d}v_y \int_{-\infty}^{\infty}\exp\left(-\frac{mv_z^2}{2kT}\right)\mathrm{d}v_z\ . \quad (11)$$

Solving these integrals using known integral

$$\int_{-\infty}^{\infty}\exp(Cx^2)\mathrm{d}x = \sqrt{\frac{\pi}{C}} \quad (12)$$

we get

$$J_x = \frac{2qm^3}{h^3}\exp\left(\frac{E_F}{kT}\right)\exp\left(-\frac{U}{kT}\right)\frac{kT}{m}\sqrt{\frac{2\pi kT}{m}}\sqrt{\frac{2\pi kT}{m}}\ ,$$

$$J_x = \frac{4\pi qmk^2}{h^3}T^2\exp\left(\frac{E_F - U}{kT}\right) \quad (13)$$

or shorter

$$J = A_R T^2 \exp\left(-\frac{W}{kT}\right), \quad (14)$$

Child's law

When thermionic electrons are emitted from the metal surface we would like to detect them. For that we use the evaporating metal as a cathode and anode at a distance $x = b$ so that they are at a potential difference V_A (figure 2.2). This potential attracts electrons from the cathode to the anode, so the electrons accelerate towards anode where they can be detected. Electric current I_A flows from anode to the cathode and the same holds for the current density J_A [A/m^2].

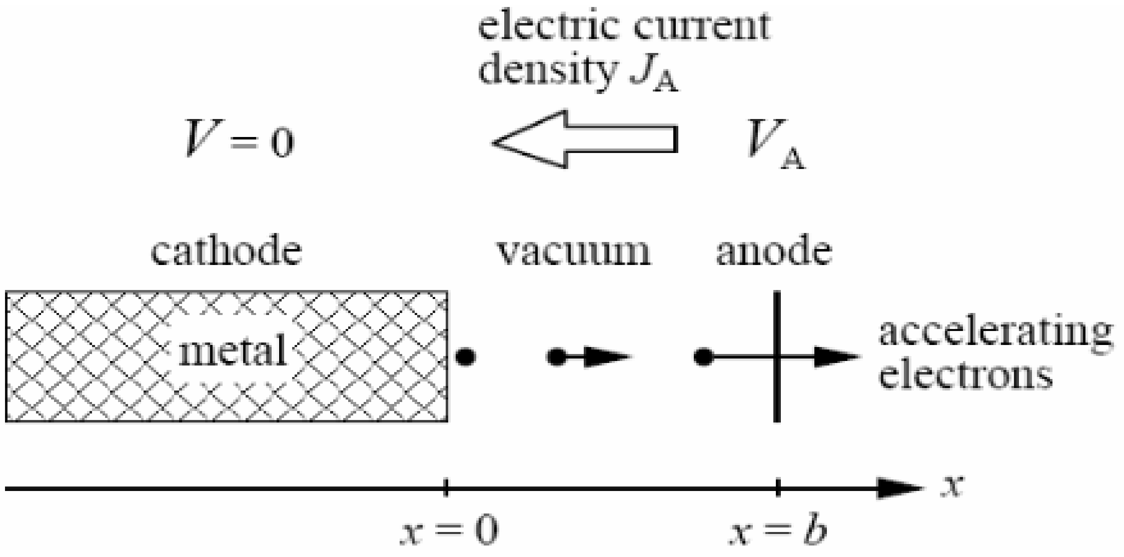

Figure 2.2: Simplified model of cathode-anode apparatus.

Let us suppose we have the setup shown on figure 2.2 in a vacuum. A cloud of thermically escaped electrons (as shown in chapter 2.1) is formed on the surface of metal, so that the space around it is unevenly filled with electrons. Space charge density (number density) of electrons $n(x)$ [m^{-3}] vary with x. Positive potential on $x = b$ pulls these electrons towards the anode. Electric current density of electrons is

$$J = -e\,n(x)\,v(x) = -J_A. \tag{15}$$

Kinetic energy of electron equals to

$$\frac{1}{2}mv^2 = eV(x). \tag{16}$$

Knowing the $V(x)$ we can calculate velocity of electrons as the function of distance x:

$$v = v(x) = \sqrt{\frac{2e}{m}V(x)}. \tag{17}$$

Accelerating electrons constitute a steady current J_A, so we can see that $n(x)$ is decreasing and $v(x)$ is increasing towards anode.

We would like to know how space charge density affects electric potential between cathode and anode. For that we can use Poisson's equation

$$\frac{\partial^2 V}{\partial x^2} = -\frac{\rho(x)}{\varepsilon_0} \tag{18}$$

that tells us how the potential $V(x)$ changes in presence of charge density $\rho(x)$. In our case

$$\frac{\partial^2 V}{\partial x^2} = \frac{en(x)}{\varepsilon_0} = \frac{J_A}{\varepsilon_0 v(x)} = \frac{J_A}{\varepsilon_0 \sqrt{\frac{2e}{m} V(x)}}. \tag{19}$$

Equation have to obey initial condition:

$$\left.\frac{\partial V}{\partial x}\right|_{x=0} = 0,$$

$$\left.V\right|_{x=0} = 0. \tag{20}$$

Solving differential equation we get the potential for every point in region $0 < x < b$:

$$V(x) = \left[\frac{9J_A}{4\varepsilon_0 \sqrt{\frac{2e}{m}}} \right]^{\frac{2}{3}} x^{\frac{4}{3}} . \tag{21}$$

We know that at point $x = b$, the potential is $V(b) = V_A$. So from equation (21) current density can be expressed as

$$J_A = \left[\frac{4\varepsilon_0}{9b^2} \sqrt{\frac{2e}{m}} \right] V_A^{\frac{3}{2}} . \tag{22}$$

Equation (22) describes the current density flow J_A at any given voltage V_A. The result is known as Child's law or Child-Langmuir[3] law.

This relation can be used for experimental determination of specific charge e/m of an electron. (See third chapter for details).

Small correction should be made here. Electrons are not emitted by the filament with zero initial velocity. They have thermal energy $\frac{3}{2}kT$ per electron. That potential difference should be added to the plate voltage, and is about 0,3 V at $T = 2000$ K.