# M C A

## PART - I

## SUBJECT
## DISCRETE MATHEMATICS

## PAPER - III

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# PROPOSITIONAL CALCULUS

**Connections and Compound Statements**

**Combination of Connection**

**Converse, Inverse and Contrapositive**

## Connectives, Compound Propositions :

The Proposition which are constructed by combing two or more simple statements with the use of certain words known as connectives. The new Propositions thus formed are called molecular or compound Propositions.

The following table gives the various connectives, their symbols and the name of the compound propositions.

| S. No. | Connective | Symbol | Name of Compound Proposition |
|---|---|---|---|
| 1. | "not" | | Negation |
| 2. | "and" | | Conjunction |
| 3. | "or" | | Disjunction |
| 4. | "If ....... then" | | Conditional or Implication |
| 5. | "If and only If" | | Bi-Conditional |

Except the first i.e. not, all he other connectives connect two or more statements to give a third compound statement.

## Negation

The negation of a statement is generally formed by introducing the word "not" at a proper place in the proposition or by prefixing the proposition with the phrase "It is not the case that"

If p is any given proposition, then the negation of "p" is written as "  p" and read as "not p". If the truth value of p is T, then the truth value of" "  p" is F and vice versa. This fact is shown in the truth table below

## <u>Truth table for Negation</u>

| p | p |
|---|---|
| T | F |
| F | T |

**Example** p : Manipal is a city,  p ; manipal is not a city. (or) It is not the case that manipal is a city.

q : 2 is less than 7

 p : 2 is not less than 7

(or) 2 is greater than or equal to 7

## Conjunction :

The Conjunction of two Propositional p and q is the proposition p  q which is read as "p and q". The Proposition p  q has the truth value T whenever both p and q have the truth value T, otherwise it has the truth value F. This fact is defined by table.

## Truth table for conjunction

| p | q | p  q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

**Note :** The truth value of p  q is same as truth value q  p.

**Example** (1) p : Roses are red

(2) q : violets are blue

p  q : Roses are red and violets are blue.

(2) p : A square is a rectangle

q : A rectangle is a quadrilateral.

p  q : A square is a rectangle and a rectangle is a quadrilateral.

(3) p : 3 + 3 = 6

q : 7 is divisible by 4

p  q : 3 + 3 = 6 and 7 is divisible by 4.

In the above example p, q are simple propositions, but the Proposition p  q is compound proposition.

**Example (1) and (2) are a true compound Proposition. Where as Example (3) is a false compound Proposition.**

## Disjunction

The disjunction of two statements p and q is the proposition p  q which is read as ‘p or q”. The proposition p  q has the truth table F only when p and q have the truth value F, otherwise it is true.

## <u>Truth table for Disjunction</u>

| p | q | p  q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

**Note :** The truth value of p  q is the same as that of q  p.

**Example** :

(1) p : Every Square is a rectangle

q : Roses are red

p  q : Every square is a rectangle or roses are red.

(2) p : 3 is a Prime number

q : 8 is a composite number

p  q : 3 is a Prime number or 8 is a composite number

In the disjunction, the connective or is used in the inclusive sense that is p  q means p or q or both.

The connectives , of logic are analogous to the symbol , in the set theory.

The Exclusive disjunction of two Propositions p and q is the statement “Either p is true or q is true, but both are not true” We denote this by p p  q.

**For Example :**

p : 2 + 3 = 5

q : Radha is an engineer.

Then p  q is the statement Either, 2 + 3 = 5 or Radha is an engineer. This will be true only if Radha is not an engineer.

## Conditional or Implication :

For any two given propositions p and q, the proposition if p then q, denoted by p  q is called the conditional or implication. This means “p implies q”

The Proposition p  q has the truth value F when q has the truth value F and p the truth value T, otherwise it has the truth value T.

## Truth table for Conditional

| p | q | p  q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

**Example**

(1) p : Two straight lines interesect

q : The opposite angles are equal.

p q : If two straight lines interesect, then the opposite angles are equal.

(2) p : 2 + 3 = 5

q : 5 is a composite number

p q : If 2 + 3 = 5, then 5 is a composite number

(3) r : I study well

s : I shall pass the Examination

r s : If I study well, then I shall Pass the Examination

**Note :** The truth value of p q is not the same as that of q p.

In the Implication p q.

p is called the antecedent or hypothesis.

q is called the consequent or conclusion.

p q meanse, if p then q or p Implies q or q provided p or p is sufficient condition for q or q is necessary condition for p.

## Bi-conditional :

A compound statement of the form [(p q) (q p)] is called a bi-conditional which is denoted by p q. It is clear from the definition that a bi-conditional is of the form "if p, then q and if q then p". Equivalently it can be put in the form "p if and only if q".

## <u>Truth table for Biconditinal</u>

| p | q | p q | q p | (p q) (q p) or p q |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

Thus p q is true when p and q are both true or both false.

(a) **Example :**

p : Two lines are perpendicular

q : They intersect at right angles.

p q : two lines are prependicular if and only if they intersect at right angles.

(b) **Example :**

p : An integer n is divisibile by 2

q : The integer n is even

p q An Integer n is divisible by 2 if and only if it is even.

## Combination of Connectives :

A compound statement can now be formed by the combination of , , , and
.

**Example :** If I study well and question papers are not difficult, then I shall pass the Examination.

The example can be symbolised as follows

r : I study well

s : Question papers are difficult

t : I shall pass the examination

We have $(r \quad s) \quad t$

**Example (2)** Write the given statement in the symbolic form : if ABC is an Isosceles triangle then its base angles are equal and the sides about these angles are also equal.

**Solution**

p : ABC is an Isosceles triangle

q : Its base angles are equal

r : The sides about these angles are equal.

Hence the given statement is $p \quad (q \quad r)$

(3) Write the truth table for $p \quad (q \quad r)$.

Truth table for $p \quad (q \quad r)$ is

| p | q | r | q   r | p   (q   r) |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | F | F |
| F | T | T | T | F |
| F | T | F | T | F |
| F | F | T | T | F |
| F | F | F | F | F |

### 1.8.1 Converse, Inverse and contrapositive of an Implication :

Let $p \quad q$ be a condition, then

(i) q p is called the converse of p q

(ii) $p$ q is called the inverse of p q

(iii) q p is called the contrapositive of p q

(a) **Example :**

If x = 3 then $x^2 + x - 12 = 0$

The given example can be symbolised

as follows

p : x = 3

q : $x^2 + x - 12 = 0$

Therefore p q

Converse : q p : if $x^2 + x - 12 = 0$ then x = 3

Inverse ( p q) : if x 3 then

$x^2 \quad x \quad 12 \quad 0$

Contrapositive : ( q p) : If $x^2 \quad x - 12 \quad 0$

then $x$ 3.

(b) Example : A student who studies well, will pass

This statement can be put in the form "If a student studies well,

Then he will pass"

This can be symbolised as follows

p : A student studies well

q : He will Pass

Therefore p q

Converse : q   p : If a student passes, then he studies well.

Inverse : (  p    q) If a student does not study well, then he will not pass.

Contrapositive : (  q    p) : If a student does not pass, then he does not study well.

Truth table of an Implication, its converse, Inverse and its contrapositive as follows :

| p | q | p | q | p  q | q  p | p  q | q  p |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | T | T | T |
| T | F | F | T | F | T | T | F |
| F | T | T | F | T | F | F | T |
| F | F | T | T | T | T | T | T |