M C A

PART - I

SUBJECT
DISCRETE MATHEMATICS

PAPER - III

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PROPOSITIONAL CALCULUS

Objective

Introduction

Definition of Statement of Proposition

Proposional Function

Truth Value and Truth Table

The Objective

After reading this unit, you should be able to

(a) Distinguish between Propositional and Non Propositional

(b) Construct the truth table of any compound Proposition

(c) Identify and use logically equivalent statements.

(d) Identify and use logical quantifiers. One of the important aim of mathematical logical M to provide rules.

Introduction

The major development in the study of logic, its formalization in terms of mathematics by Leibniz (1646–1716) and George Boole (1915–1864) who seriously studied and developed this theory, called symbolic logic. M the Introduction to the block you have read about what symbolic logic is using it we can formulate, so our arguments and logical reasoning in a manner that can easily show if the reasoning is valid or is a finally. We talk about which kind of sentences is acceptable in mathematical logic. We call such sentence statements or Propositions. You will also see that a statement can either be true or false. Accordingly, as you will see, we will give the statement a truth value T or false F.

We begin our study of the logical relationship between Propositions. This is called, Propositional calculus. In this we look at some ways of connecting simple Propositions to objection more complex one. To do so, we use logical connections like “and” and “or” We also Introduce you to other connections like “not”, ‘Implies’ and ‘implies and is Implied by’. At the same time we construct tables that allow us to find the truth values of the compound statement that we get.

In this unit we consider the conditions under which two statements are “The same’. In such a situation we can safely replace one by the other.

In this unit we talk about some common terminology and Notation which is useful for quantifying the objects which we are dealing with in a statement.

Propositions or Statements

A Declarative sentence which is either ‘true’ or ‘false’ but not both at the same time is called a Proposition or statement.

Examples :–

(I) India is Country

(II) $5 + 5 = 10$

(III) $\sqrt{2}$ is an Irrational number

(IV) Bangalore is the capital of India

(V) The sum of two even integers is an odd.

(VI) Every Interger is divisibe by 3.

All the above sentences are Propositions. The first three are true statements, where as the last three are false statements.

Note : That the following are not statements

(VII) 5x is a natural number.

(VIII) In which state are you staying ?

(IX) Who are you ?

(x) He is a cricket Player.

Sentence (VII) is not a statement because x is not described there, where as (VIII)) and (IX) not a declarative at all and Questions. In (X) sentence we do not know who he is.

The above give examples from 1 to (vi) are called atomic or primary or simple statement. Which are denoted by the letters p, q, r, s etc.

For Example

p : India is a country.

Proposional Functions :

Just as conventional calculus, in mathematics, deals with functions showing relationship between quantities expressed in terms of variables within a valid domain, Propositional calculus deals with proposional functions comprising statements. It deals with the structure of sentences as composed of components of sentences. Variables are used to express such components. rather than quantities. A normal sentence comprises of a subject and a Predicate. A predicate is that component of a sentence, as a word or combination of words, which expresses the nature of a subeject. Components of a sentence are joined by logical connectives in propositional functions. In a statement, 'x is a natural number', The Predicate is 'a natural number' which tells about the nature of x. In a statement like p(x); 'x is a natural number' is called an open sentence.

An expression or statement, 'x is mortal' can be converted into a proposional by replacing 'x' with a determinate value. If we assign a value to 'x' as Aristotle, then it becomes a proposition. 'Aristotle in mortal'. This term originated in an attempt to derive mathematics from logical axioms. Propositional function is also known as a "statement function"

1.4 Proposition according to mathematics; is a statement or theorem, usually containing its proof. It is a statement that affirms or denies something. As we know that any formulae that we use, contains sysmbols as variables and operators that define operations to be performed. This is also true for Propositional functions. In a proposition, a Boolean valued function is used which is a function of the type f : X B. Here X is any arbitrary set and B is a Boolean domain.

A Boolean domain B is a set of 2 discrete elements. For example B = {0, 1}. Elements of such a domain are interpreted as logical values for Example O = False and 1 = True. This logic is followed in many applications. Discrete mathematics uses this principle as a tool to be used by other fields too, like business and scientific applications. For example, o may represent absence of a signal and 1 may denote the presence of the signal. This can be applied in the field of tele communication.

A Propositional function is an expression containing symbols for its variable elements and becomes a proposition when values are substituted for symbols. Propositional formula or a propositional expression or Propositional function a sentence or a sentential formula is a syntactic expression which is formed from the elements of a given alphabet A of Propositional variables, for Example A = {p, q, r, ...), together with the elements of a given set of operator symbols, form propositional logic. Such operating symbols are k-adic operator symbol like the o adic operator symbol {false, true}, the 1-adic symbol , and some sub sets of the 2 adic symbols in { , , , }. other collections of operator symbols are also used, which depends on that particular formal language used for the purpose.

A Propositional variables takes Boolean value either True or False and logical symbols. such variables are the basic building blocks of Propositional formulae or functional and thus these propositional variables are used in propositional and higher logics.

Truth Value and Truth Tables

The truth or falsity of a given Proposition is called its truth value, If a Proposition is true, it is denoted by ‘T’ and if false, it is denoted by ‘F’.

A truth table presents compactly the truth values of one or more propositions for all the possible combinations of the truth values of each primary propositions.