B.C.A -PART - I

SUBJECT : MATHEMATICS

PAPER-IV

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UNIT 2 DETERMINANTS

2.1 Introduction :

Theory of determinants was first studied in course of solving a system of linear equations. Now a days it occupies a significant place in science, social science, computer science and several other branches.

Consider two linear equation in 2 variables x and y

$$\left.\begin{array}{l} a_1x+b_1y=0 \\ a_2x+b_2y=0 \end{array}\right\} \quad \text{....(2.1)}$$

Eliminating x and : $\frac{x}{y}=\frac{-b_1}{a_1}=\frac{-b_2}{a_2}$

This gives $a_1b_2-a_2b_1=0$

We can write it as $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=0$

Where $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=a_1b_2-a_2b_1$ ...(2.2)

Similarly, eliminating the variables x an y from th equations

$$\left.\begin{array}{l} a_1x+b_1y+c_1z=0 \\ a_2x+b_2y+c_2z=0 \\ a_3x+b_3+c_3z=0 \end{array}\right\} \quad \text{....(2.3)}$$

We shall obtain

$$a_1(b_2c_3-b_3c_2)-b_1(a_2c_3-a_3c_2)+c_1(a_2b_3-a_3b_2)=0$$

i.e. $a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}-b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}+c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}=0$

We can write it as $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

where

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1\left(b_3c_3 - c_2b_3\right) - b_1\left(a_2c_3 - c_2c_3\right) + c_1\left(a_2b_3 - b_2a_3\right) \qquad .....(2.4)$$

2.2. Definition of a Determinants

If a, b, c, d are four numbers, we say that

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \text{a Determinants of order 2}$$

It has two horizontal rows and two vertical columns

We define its value as

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a.d - b.c$$

Ex. (i) Evaluate $\begin{vmatrix} 3 & 4 \\ 5 & 9 \end{vmatrix}$

Ans. $\because \begin{vmatrix} 3 & 4 \\ 5 & 9 \end{vmatrix} = 3 \times 9 - 4 \times 5 = 27 - 20 = 7$

Ex. (ii) Evaluate $\begin{vmatrix} 3x & 5x+2 \\ 2 & 4x-1 \end{vmatrix}$

Ans. $\because \begin{vmatrix} 3x & 5x+2 \\ 2 & 4x-1 \end{vmatrix} = 3x\left(4x-1\right) - 2\left(5x+2\right) = 12x^2 - 13x - 4$

Next, If $a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3$, are $3^2 = 9$ numbers, we say that

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_3 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \text{a determinant of order .3}$$

It has 3 (horizontal) rows and 3 (vertical) Columns

Similarly, we can define a determinant of order n. It will have n rows and n columns

A determiants has always equal number of rows and columns.

In our context, we are concerned with the determinants of order 2 and 3. So we restrict our attention to these determinants alone. We sometimes denote a determinant by the symbol Δ.

2.3. Rule to put Proper Sign Before an Element :

We Consider a determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad ....(2.5)$$

We use double suffix for an element in his determinants, when the first symbol stands for position of its row and second suffix for position of its column

Ex. a_{11} = the element occuring in first row and first column

a_{23} = the element occuring in second row and third columns

In general,

$a_{ij} = (i, j)$ the element = element occuuring in ithrow and jth column for proper sign (+ or –) before an element a_{ij}, we consider the sum of positions of its row and column (ie i + j). in the determinant Δ.If this sum is even, we put +ve sign before the element and if this sum is odd, we put a – ve sign before the element

In fact, for sign before the element a_{ij}, we consider the quantity, where i and j denote the position of its row and column respectively in the determinants Δ.

Since a_{11} occurs in first row and first column and $(-1)^{1+1} =)(-1)^2 = +$ therefore +ve sign occurs before a_{11}.

Also, since a_{12} occurs in first row and second column and $(-1)^{1+2} = (-1)^3 = -$, therefore -ve sign occurs before a_{12}.

Set the proper sign of an element for its particulars place in the determinants Δ. can be expressed as

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

We note that the diagonal containing the elements a_{11}, a_{22}, a_{33} in the determinant Δ. is called the principal diagonal

2.4. Minor and Cofactor of an Elements :

Let $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{23} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

We define

Minor of an element.

= The determinant obtained from Δ on deleting

or removing the row and the column in which this element occurs

Ex. $M_{11} = \text{Minor}(a_{11}) = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$

$M_{12} = \text{Minor}(a_{12}) = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = a_{21}a_{33} - a_{23}a_{31}$

Next, we define

Cofactor of an elements

= (Proper sign of the elements) (Minor of the element)

Hence $c_{ij} = \text{cofactor}(a_{ij}) = (-1)^{i+j}.\text{Minor}(a_{ij})$

Ex. c_{11} = cofactor $(a_{11}) = (+)\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$

c_{23} = cofactor $(a_{23}) = (-)\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = (a_{11}a_{32} - a_{12}a_{31})$ etc.

Q. Find the minors and cofactor of 1, 6 and 8 in

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Ans. We have M_{11} = $\text{Minor}(1) = \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = 5 \times 9 - 6 \times 8 = -3$

$M_{23} = \text{Minor (6)} = \begin{vmatrix} 1 & 2 \\ 2 & 8 \end{vmatrix} = 1 \times 8 - 2 \times 7 = -6$

M_{32} = Minor (8) $\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = 1 \times 6 - 3 \times 4 = -6$

Also c_{11} = cofactor (1) = (+) M_{11} = – 3

c_{23} = cofactor (6) = (+) M_{23} = + 6

c_{32} = cofactor (8) = (+) M_{32} = + 6

2.5. Expansion of a Determinant :

The value of a determant Δ is the sum of products of the elements in any row or column with its corresponding cofactors.

As an illustration, see (2.4)

So we can expand a determinant by any one of its row or column

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

First, we expand the determinants Δ by its first row as

$\Delta = a_1$(its cofactor) + b_1 (its cofactor) + c_1 (its cofactor)

= a_1 (+) (its minor) + b_1(–) (its minor) + c_1 (+) (its minor)

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_2 - a_3c_3) + c_1(a_2b_3 - a_3b_2)$$

Expansion by first row

We note that, in any determinant Δ

its row expansion = its column expansion

Q. Evaluate $\begin{vmatrix} 1 & 3 & -2 \\ -2 & 4 & 1 \\ 3 & -1 & 2 \end{vmatrix}$

Ans. $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ -2 & 4 & 1 \\ 3 & -1 & 2 \end{vmatrix}$ Expanding by first row

$$=1\begin{vmatrix}4 & 1\\ -1 & 2\end{vmatrix}-3\begin{vmatrix}-2 & 1\\ 3 & 2\end{vmatrix}+(-2)\begin{vmatrix}-2 & 4\\ 3 & -1\end{vmatrix}$$

$$=(8+1)-3(-4-3)-2(2-12)$$

$$=9-21+20=8$$

Note : In a determinant Δ of order 3.

as given in (2.5), we denote its first row, second row and third row R_1, R_2 and R_3 respectively). Also we denote its first column, second column and third column by C_1, C_2 and C_3 respectively.

2.6. Properties and Determinants :

There are some important properties of determinats, which are required simplify it.

2.6.1 If rows and columns of a determinants are interchange, there is no change in its value

Let $\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$

Interchanging its rows and columns (i.e. $R \leftrightarrow C$)

$$\Delta^1=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

Then $\Delta^1=\Delta$

for, $\Delta=a_1.A_1+b_1\ B_1+c_1.C_1$

(expanding by its first row)

Here A_1, B_1, C_1 denote cofactors of a_1, b_1, c_1 respectively

Also, $\Delta^1=a_1A_1+b_1B_1+c_1.C_1$

so, $\Delta'=\Delta$ (Expanding Δ' by first column)

2.6.2. If any two adjacant rows (or columns) of a determinant are interchanged, the sign of the determinant is changed, but its absolute value remains unaltered.

Let. $\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$

$= a_1 (b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$

(expanding by first row)

If its first row and second row are interchanged (i.e. $R_1 \leftrightarrow R_2$)

$$\Delta' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

then $\Delta' = -\Delta$

for, $\Delta' = a_2(b_1c_3 - c_1b_3) - b_2(a_1c_3 - c_1a_3) + c_2(a_1b_3 - b_1a_3)$

$= -\{a_1(b_2c_3 - c_2b_3)b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)\}$

$= -\Delta$

2.6.3: If any two paricular rows (or columns) or a determinant are same, its value is zero

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Where first row and second row are same (i.e. $R_1 = R_2$)

When first row and scond row are interchanged, then by the last property, its value will be $-\Delta$. since the first row and second row are same the value of the determinant will be Δ.

So, $\Delta = -\Delta$ i.e. $2\Delta = 0$ i.e. $\Delta = 0$

2.6.4. If every element of a row (or column) of a determinant is determinatn is multiplied by a non-zero constant m the determinant itself is multipled by m

$$\text{Let } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Multiplying its first row by m (i.e. applying $m \times R$)

$$\text{Let } \Delta^1 = \begin{vmatrix} ma_1 & mb_1 & mc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

then $\Delta^1 = m.\Delta$

For, $\Delta^1 = (ma_1)(b_2c_3 - c_2b_3) - (mb_1)(a_3c_3 - c_2a_3) + (mc_1)(a_2b_3 - b_2a_3)$

$= m\{a_1(b_2c_1 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)\}$

$= m.\Delta$

2.6.5 If each element of any particular row (or column) is the sum of two or more terms, then it determinant can be expressed as the sun of two or more determinants of this same order.

the implies that

$$\begin{vmatrix} a_1 + l_1 + m_1 & b_1 & c_1 \\ a_2 + l_2 + m_2 & b_2 & c_3 \\ a_3 + l_3 + m_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} l_1 & b_1 & c_1 \\ l_2 & b_2 & c_2 \\ l_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} m_1 & b_1 & c_1 \\ m_2 & b_2 & c_2 \\ m_3 & b_3 & c_3 \end{vmatrix}$$

It can be easily verified by expanding the given determinant by its first column.

2.6.6 If each element of any particular row (or column) is increased or decreased by some constant multiple of the corresponding element of another row (or column). then the value of the determinant remains the same

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Multiply its second row by m and then adding it to first row (i.e.applying R_1 +m)

$$R_2 \to R_1) \Delta = \begin{vmatrix} a_1 + ma_2 & b_1 + mb_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

then $\Delta^1 = \Delta$

It can be easily verified by expanding the determinant Δ^1 by first row.

2.6.7 The value of the following determinant is equal to products of its elements in principal diagonal this implies that.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ 0 & b_1 & c_2 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1.b_2.c_3$$

Expanding first determinants by first column

and second determinants by first column the above result can be checked.

2.7 How to Simplify a Determinant :

First, we make use of suitable properties of a determinant Δ (of order 3) to bring two zeros in any row (or column) of it. Next, we expand the determinant Δ by that row (or column) containing zero. Then the determinant Δ will be reduced to a determinants of order 2, whose value can be easily found.

Ex. $\begin{vmatrix} a_1 & 0 & 0 \\ a_1 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (there are two zeros in R_1)

Expanding by first row,

$$\Delta = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = a_1\left(b_2c_3 - c_2b_3\right)$$

2.8 Cramer's Rule for Solving a System of Linear Equations :

2.8.1. System of two Linear equations in two veriables

Consider a system of two linear equation in two variables x and y as

$$\left.\begin{aligned} a_1x + b_1y = d_1 \\ \text{and} \quad a_2x + b_2y = d_2 \end{aligned}\right\} \qquad ...(2.6)$$

Where $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_2b_2 - a_2b_1 \neq 0$

This gives

$$a_1x + b_1y - d_1 = 0$$
and
$$a_2 + x + b_2y - d_2 = 0$$

Applying cross-multiplication method.

$$\frac{x}{-b_1d_2 + b_2d_1} = \frac{y}{-a_2d_1 + a_1d_2} = \frac{1}{a_1b_2 - a_2b_1}$$

i.e. $\dfrac{x}{\begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}} = \dfrac{y}{\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$

This gives $\dfrac{x}{\Delta x} = \dfrac{y}{\Delta y} = \dfrac{1}{\Delta}$ ....(2.7)

where $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \Delta_x = \begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}, \Delta_y = \begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}$

This gives the solution of the system of equation....(2.6)

It can also be written a,

$$x = \frac{\Delta x}{\Delta}, y = \frac{\Delta y}{\Delta} \qquad ....(2.8)$$

2.8.2 Sytem of three linear equation in three variables

Consider a system of three linear equation in three variables x, y and z as

$$\left.\begin{aligned} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{aligned}\right\} \qquad ...(2.9)$$

$$\text{where } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$

Δ is known as the determinant of coefficients

$$\text{Take } \Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Here Δ_x is obtained on removing first column of Δ and then replacing a_1, a_2, a_3 by $d_1d_2d_3$ respectively.

Also Δy is obtained on removing second column of Δ and then replacing b_1, b_2, b_3, by $d_1d_2d_3$ respectively.

Similarly, Δ_z is obtained on removing third column of Δ and then replacing c_1,c_2,c_3 by d_1, d_2, d_3 respectively.

Then according to Cramer's Rule, the solution of the given system of equations (2.9) is given by

$$\frac{x}{\Delta_x} = \frac{y}{\Delta_y} = \frac{z}{\Delta_z} = \frac{1}{\Delta} \qquad .... (2.10)$$

This can also be wirtten as

$$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta} \qquad ....(2.11)$$

2.8.3 Deductions from Cramer's Rule

A. The following cases arises for solution of the system of equations (2.9)

Case 1: When $\Delta \neq 0$, then the system of equations (2.9) has a unique solution as given by (2.11)

Case 2: When $\Delta = 0$ and at least one of Δ_x or Δ_y or $\Delta_z \neq 0$.

Then system of equations (2.9) are said to be in consistent and it has no solution.

Case 3: When $\Delta = 0$ and $\Delta_x = \Delta_y = \Delta_z = 0$. Then the system of equation (2.9) has infinite number of solution.

B. For a system of three homogenous equations in three variables :

Consider a system of three homogenous linear equations in three varables x y and z as

$$\left.\begin{aligned} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \\ a_3 + b_3y + c_3z = 0 \end{aligned}\right\} \qquad ....(2.12)$$

The system of equations (2.12) will have non-trivial (i.e. non-zero) solution, if

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

In this case, we have $\Delta_x = \Delta_y = \Delta_z = 0$

So this system with be said to be consistent and it has infinitely many solutions.

2.9. Worked out Examples :

1. Evaluate (i) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ (ii) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Ans. for (i) Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ apply $C_1 \to C_1 - C_3 \,\&\, c_2 \to c_2 - c_3$

then $\Delta = \begin{vmatrix} 0 & 0 & 0 \\ a-c & b-c & c \\ a^2-c^2 & b^2-c^2 & c^2 \end{vmatrix}$ expand by first row

$$=1.\begin{vmatrix} a-c & b-c \\ a^2-c^2 & b^2-c^2 \end{vmatrix} = \begin{vmatrix} a-c & b-c \\ (a-c)(a+c) & (b-c)(b+c) \end{vmatrix}$$

$$= (a-c)(b-c)\begin{vmatrix} 1 & 1 \\ a+c & b+c \end{vmatrix}$$

= (a – c) (b – c) {(b + c) – (a + c)} = (a – c) (b – c) (b – a)

= {–(c – a)} (b – c) {–(a – b)} = (a – b) (b – c) (c – a)

for (ii) : Let $\Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ Taking common column wise

Ans. $\Delta = a.b.c\begin{vmatrix} 1 & 1 & c \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

= a b c (a – b) (b – c) (c – a), by part (i)

Example . Solve by Cramer's rule

2x – y + 3z = 9

x + y + z = 6

x – y + z = 2

Ans. Let $\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} \begin{pmatrix} \text{Deter min ant} \\ \text{of Coefficients} \end{pmatrix}$

Then $\Delta = 2(1+1)+1(1-1)+3(-1-1) = -2$

Since $\Delta = -2 \neq 0$, therefore the given system has a unique solution.

Also $\Delta_x = \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix}$

$= 9(+1)+1(6-2)+3(-6-2) = -2$

$\Delta_y = \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix}$

$= 2(6-2) - 9(1-1) + 3(2-6) = -4$

$$\Delta_z = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix}$$

$= 2(2+6) + 1(2-6) + 9(-1-1) = -6$

So by Cramer's rule

$x = \frac{\Delta_x}{\Delta} = \frac{-2}{-1} = 1$ **Ans.** x = 1, y = 2, z = 3

$y = \frac{\Delta_y}{\Delta} = \frac{-4}{-2} = 2$

$z = \frac{\Delta_z}{\Delta} = \frac{-6}{-2} = 3$