# B.C.A -PART - I

## SUBJECT : MATHEMATICS

## PAPER - IV

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# SET THEORY

## 1.1 Introduction

The notion of sets was first explicitly introduced and developed by G. Cantor (1845-1918). At present, set theory plays the most powerful role in all branches of science, social science computer science and Technology.

### 1.1.1 Definition and Examples :

In the universe, everything whether living or nonliving is known as an object. A given collection of objects will be called well defined, if we can definitely say whether a particular object is a member of our collection or not!

**A set is a collection of distinct and well defined objects, which are known as its elements or members.**

The member of a sct may be either a physical object (such as Book, student) or an object of thought (such as number, alphabet)

**Examples of a Set**

1. The collection of all months of a year
2. The collection of all vowels a, e, i, o, u in English alphabet.

**Examples of not a Set**

1. The collection of all intelligent students in a class. It is not a set, because the term “intelligent” is vague and it is not well-defined.
2. The collection of all beautiful girls attended in a New Years day party.

Here the term “beautiful” is also vague and it is not well defined.

**Notation**

The sets are usually denoted by capital letters in English alphabet such that A, B, C, P, Q, X etc. Also the elements of a set are denoted by small letters such as a, b, c, p, q, x, y etc or by numericals such as –3, –1, 0, 1, 3, 10 etc.

**Memebership Relation** $\in$

If an element x is a members of a set A, we write $x \in A$ and say that x belongs to A.

Also, if an element y is not a member of a set B, we write $y \notin B$ and say that y does not belong to B.

Ex. Let X = {a, b, d, p, q, x}

Then $a \in X, p \in X$ but $C \notin X, r \notin X$

### 1.1.3 Representation of a set :

There are two widely used methods of representing a set

**(i) Tabular form or Roster method**

In this form, we list all the members of the set within the middle bracket { } and seperate its members by commas.

Ex. (i) Let A = set of all the vowels in its English alphabet the five vowels are a, e, i,o, u

So A = {a, e, i, o, u} (Tabular form)

Ex. (ii) Let B = set of all factors of 24 $\equiv$

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

So B = {1, 2, 3, 4, 6,8, 12, 24} {Tabular form}

**(ii) Set-builder form or property form or Rule Method**

In this from, a set may be represented by stating a property or properties which all its elements must satisfy. If P(x) is a property which is satisfied by each member x of a set A, then we write.

A = {x : P(x)}

Ex. (i) Let N = set of all natural number

then N = {x : x is a natural number}

Ex. (ii) Let X = set of all integers from –2 to 10

then X = {x : x is integer; $-2 \le x \le 10$ }

### 1.1.4 Some standard sets and their symbols :

(i) N = set of all natural numbers

= {1, 2, 3, 4,-------,n-------}

(ii) 1 or Z = set of all integers

= {0, $\pm 1$, $\pm 2$, $\pm 3, \ldots\ldots \pm n \ldots\ldots$}

(iii) Q = set of all rational numbers

= { $\frac{p}{q}: p, q$ are integers : $q \neq 0$ }

So all fractions such as $\frac{1}{2}, \frac{3}{4}, \frac{-4}{9}$ etc. and all integers such as –2, 0, 1, 5 etc are included in Q.

(iv)R = set of all real numbers

= {x : x is real ; $-\infty < x < +\infty$ }

$= \{-\infty + \infty\}$ or $]-\infty, +\infty[$

(v) C = set of all complex numbers

= {z : z is a complex number}

= $\{z : z = x + iy$ or $(x, y);\ x, y \in R : i = \sqrt{-1}\}$

**1.1.5. Classification of Sets :**

**1. Empty set of Null set**

A set having no element in it, is called an empty set. this set is usually denoted by $\phi$.

Ex (i) Let A = {x:x R; $x^2$ + 1 = 0}

Since there is no real number satisfying the equation $x^2 + 1 = 0$, therefore A = $\phi$.

Ex (ii) Let B = {x : x = an integerm: $4x^2 - 1 = 0$ }

Since $4x^2 - 1 = 0$ gives $x^2 = \frac{1}{4}$, so that $x = \pm\frac{1}{2}$: which are not integer, therefore B=$\phi$.

**2. Singleton set**

A set having only one element, is called a singleton set

Ex. A = {a}

**3. Finite set and Infinite set**

If number of elements in a set is finite, the set is called a finite set. So the process of counting the elements of a finite set comes to an end.

If number of distinct elements in a finite Set A is K, we write n(A) = K

Ex (i) Let A = set of all the months in a year

Here n(A) = 12 = finite. So the set A is finite

Ex (ii) Let B = {x : x is integer; $0 \le x \le 10$ }

Here n(B) = 11 = finite. So the set B is also finite.

Next, if the number of elements in a set is infinite the set is called an infinite set. So the process of counting elements of an infinite set soes not come to an end.

Ex (i) Let N = set of all natural numbers

= {1, 2, 3, 4,-----,n,------}

N has infinite number of elements. So N is an infinite set.

Ex (ii) Let X = set of all points in a line segment goning two points A and B

Since there are infinite number of points on the line segment between A and B, therefore X is an infinite set.

**4. Equal sets**

Two finite sets A and B are said to be equal, written as A = B, if A and B have exactly the same elements.

Ex. Let $A = \left\{ \frac{2}{4}, \frac{3}{9}, \frac{2}{8} \right\}$ and $B = \left\{ \frac{3}{6}, \frac{4}{12}, \frac{8}{32} \right\}$

then A = B $= \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\}$ $\therefore A = B$

**5. Equivalent Sets**

Two finite sets A and B are said to be equivalent written as A ~ B, if A and B have the same number of elements (i.e.n(A)=n(B))

Ex let A = {1, 3, 5, 7, 9} and B = {2, 4, 6, 8, 10}

then n (A) $\equiv$ n(B) = 5

So A and B are equivalent sets, but $A \neq B$

6. Family of sets or Set of sets

A set, whose every member is itself a set, is called a family of Sets

Ex. (i) S {{1}, {3, 4},{5,6,7}, {8}}

Here every member of S is a set. So S is a family of sets

Ex. (ii) T = {1,2, {3,4},{6}

**1.4. Different types of sets :**

**1. Subset and Superset**

Let A and B be any two sets. If every element of A is also an element of B, A is said to be a subset of B. It is denoted by $A \subseteq B$.

The symbol '$\subseteq$'is used to represent "a subset of".

So A is a subset of B,

Written as $A \subseteq B$, if

for all $x \in A \Rightarrow x \in B$

If $A \subseteq B$, then B is called a superset of A and we can write $B \supseteq A$.

Ex. (i) Let A = {1,2,4,5} and B={0, 1, 2, 3, 4, 5, 6}

then every element of A is an element of B.

So $A \subseteq B$

**2. Proper Subset**

If A and B be two sets such that every element of A is an element of B and B contains at least one element which does not belong to A, then A is called a proper subset of B. It is denoted by $A \subset B$ .

So if $A \subseteq B$, but $A \neq B$, then A is called a proper subset of B, written As $A \subset B$.

Ex. Let $A = \{1,3,5\}$ and $B = \{1,2,3,4,5\}$

then $A \subseteq B$ but $A \neq B$. So $A \subset B$

**Properties of Subsets**

(i) Empty set $\phi$ is a subset of every set A (ie $\phi \subseteq A$ )

(ii) Every set A is a subset of itself $(\text{i.e.} A \subseteq A)$

(iii) If $A \subseteq B$ and $B \subseteq A$ then A = B

(iv)If a set A has n distinct elements, then number of subsets of A is $2^a$

For, take A = $\{a_1, a_2, a_3, \ldots, \ldots a_n\}$

then A has n distinct elements we have

number of subsets having no element $1 = n_{C_0}$

number of subsets having 1 element $= n_{C_1}$

number of subsets having 2 elements $= n_{C_2}$

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number of subsets having nelements $= n_{C_n}$

Adding all these, we obtain

Total number of subsets A

$$= n_{C_n} + n_{C_1} + n_{C_2} + \ldots . + n_{C_n} = 2^n$$

$$\left(Q\, n_{C_0} + n_{C_1} + n_{C_2} + \ldots .. + n_{C_n} = 2^n\right)$$

Ex. Find all subsets of the set A= {a,b,c}

As All subsets of A are

$\phi$,{a}, {b}, {c}, (a,b}, {b, c}, {c,a}, {a,b,c}

Also number of subsets of A = $2^3$ = 8

**3. Equality of sets**

Two sets A and B are said to be equal, written as A = B, if every element of A is an element of B. and every element of B is an element of A.

i.e. if $A \subseteq B$ and $B \subseteq A$

Hence A = B if $A \subseteq B$ and $B \subseteq A$

i.e. if for all $x \in A \Leftrightarrow x \in B$

**4. Universal set**

If all the sets in our context are subsets set X, then X is called an universal set in this context. So the universal set X is the superset of all other sets in our discussion.

This set is usually denoted by U or $\Omega$ or X.

Ex. Let U = {1, 2,3,4,5,6,7,8,9,10}

Take A ={1,3} = B={2,4,6}, C={3, 5}, D ={9}

**5. Powerset**

If A be any set, then the power set of A, denoted of P(A), is the family of all possible subsets of A

So P(A) = power set of A = {B : $B \subseteq A$}

**Ex.** Let A = {a, b}, then

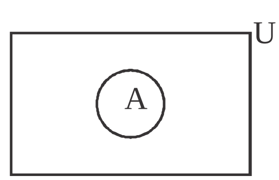
P(A)={$\phi$, A, {a},{b}}

From the property of subsets, it follows that if a set A has k distinct elements (i.e.n(A)=k)

Then n {P(A)}=$2^k$

### 1.1.7.Venn Diagram or Venn-Euler Diagram :

Venn diagram is a pictorial representation of sets and their operations. The universal set U is represented by a rectangle and if $A \subseteq U$ then the set A is usually represented by a circle within U.



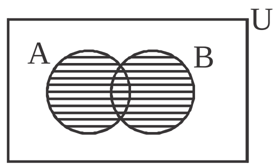

### 1.1.8.Operation of Sets :

**1. (Union of two set)**

Let A and B be any two sets, then the union of A and B, written as $A \cup B$ is a set of all such elements which are either in A or in B both.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

**Venn-diagram to represent** $A \cup B$




$A \cup B$ is shaded

Ex. Let A = {1,2,4} and B ={2,3,4,5}

then $A \cup B$ = {1,2,3,4,5}

Here We have written all the elements of A and B and their common element are written once

**Ex. (ii)** A = {x : x $\in N$; x is a factor of 8}

and $B = \{x : x \in N$; x is a factor of 12}

then A= {1,2,3,4,8} and B ={1,2,3,4,6,12}

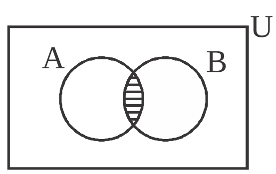
so $A \cup B$ = {1,2,3,4,6,8,12}

**2. Intersection of two sets**

Let A and B be any two sets, then the intersection of A and B, denoted by $A \cap B$, is the set of all the common elements of A and B

So $A \cap B$ = {x : x $\in A$ and $x \in B$ }

**Venn-diagram to represent A B**




$A \cap B$ is shaded

Note

Ex. (i) Let A {1,3,5,7} and B={1,3,4,5,6,9}

then $A \cap B$ = {3,5}

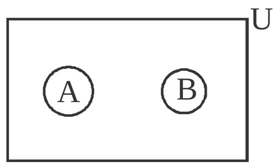
(i) $A \cap B \subseteq A$

(ii) $A \cap B \subseteq A$

**3. Disjoint Sets**

Two sets A and B are said to be disjoint, if there is no element common to both A and B ie if $A \cap B = \phi$

**Venn-diagram to represent disjoint sets**




Ex. Let A ={1,2,4} and B={3,5,7,9}

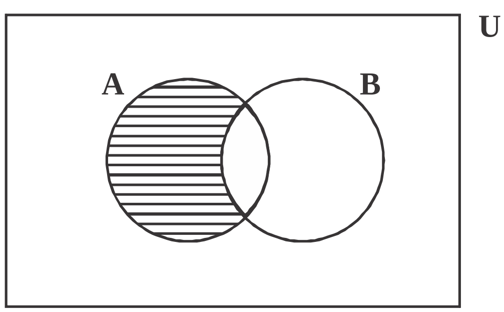
then $A \cap B = \phi$. So A and B are disjoint sets

**4. Difference of two sets**

Let A and B be any two sets, then their difference as denoted by A – B is the set of all such elements of A which are not in B.

So, A – B = $\{x : x \in A$ and $x \notin B\}$

**Venn-diagram to represent A – B**



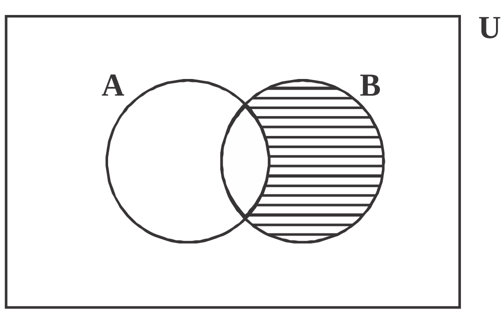

**A — B is shaded**

Similarly, we define

B–A= set of all such elements of B which are not in A

$= \{x : x \in B \text{ and } x \notin A\}$

**Venn-diagram to represent A – B**




Ex. Let A={1,2,4,5} and B={–1,2,3,4,6}

Here 2 and 4 are common elements of A and B

$\therefore A - B = \{1,5\}$ and $B - A = \{-1,3,6\}$

**5. Symmetric difference of two sets**

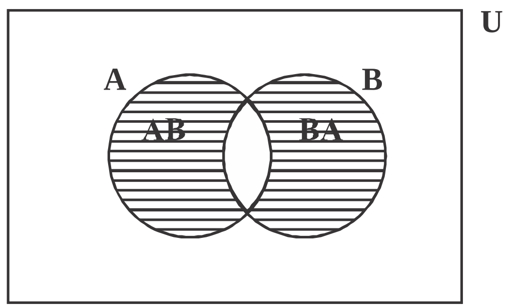
Let A and B be any two sets, the symmetric difference of A and B, denoted by $AVB$ is the set of all such elements which are either (belong to A but not in B or(belong to B but not in A)

so, $AVB$ =symmetric difference of A and B

$= \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$

$= (A - B) \cup (B - A)$

**Venn-diagram to represent** $AVB$




Let A ={1,2,3,4,5} and B={2,3,5,7,9}

Here 2,3,5 are to common elements of A and B

so A–B={1,4} and B–A={7,9}

$\therefore AVB = (A - B) \cup (B - A) = \{1,4,7,9\}$

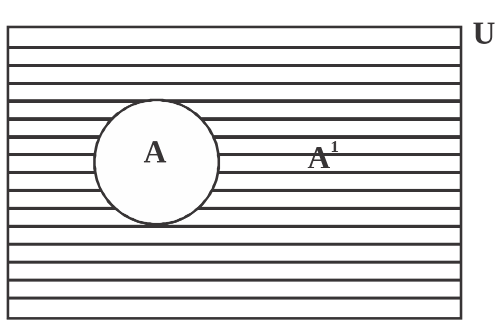
**Complement of a set**

Let U be the universal set, If $A \subseteq U$, then the complement of A in U, denoted by $\mathbf{A}^1$ is the set of all such elements of U,which are not in A

so $A^1$=complement of A in U=$A^c$

$= \{x : x \in U \text{ and } x \notin A\}$

**Venn-diagram to represent A**



Ex. Let U={1,2,3,4,5,6,7,8}

If A={1,3,5} and B={2,4,6,8}

then $A \subseteq U$ and $B \subseteq U$

so $A^1$={2,4,6,7,8} and $B^1$={1,3,5,7}

**Properties of complements**

(i) $\phi^1 = U$

(ii) $U^1 = \phi$

(iii) $\left(A^1\right)^1 = A$

(iv) $A \subseteq B \Leftrightarrow B^1 \subseteq A^1$

(v) $A - B = A \cap B^1$

For, for all $x \in A - B$

$\Leftrightarrow x \in A$ and $x \notin B$

$\Leftrightarrow x \in A$ and $x \in B^1$

$\Leftrightarrow x \in A \cap B^1$

so, by definition , $A - B = A \cap B^1$

**1.1.7 Important results for solving practical problems:**

(1) If A and B be any two finite sets, then

(i) $n\left(A \cup B\right) = n(A) + n(B) - n(A \cap B)$

(ii) $n\left(A \cap B\right) = n(A) + n(B) - n(A \cup B)$

**Particular case**

If A and B are two disjoint sets(i.e. $A \cap B = \phi$ ),then $n A \cup B = n(A) + n(B)$

(2)If A,B and C be any three finite sets, then

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

**Particular case**

If A,B and C are three mutually disjoint sets

(i.e. $A \cap B = \phi, B \cap C = \phi$ and $C \cap A = \phi$ ), then

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$