B.C.A -PART - I

SUBJECT : MATHEMATICS

PAPER - IV

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1.1.10. Algebraic Laws of Sets :

1. Identity Laws

If U be the universal set and $A \subseteq U$,then

(i). $A \cup \phi = A$ (ii) $A \cap U = A$

2. Idempotent Laws

If A be any set, then

(i) $A \cup A = A$ (ii) $A \cap A = A$

3. Commutative Laws

If A and B be any two sets, then

(i) $A \cup B = B \cup A$ (Commutative law for union)

(ii) $A \cap B = B \cap A$ (Commutative law for intersection)

4. Associative Laws

If A,B and C be any three sets, then

(i) $A \cup (B \cup C) = (A \cup B) \cup C$ (Associative law for union)

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$ (Associative law for intersection)

For (i)

for all $x \in A \cup (B \cup C)$

$\Leftrightarrow x \in A \text{ and } x \in B \cup C$

$\Leftrightarrow x \in A \text{ or } (x \in B \text{ or } x \in C)$

$\Leftrightarrow (x \in A \text{ or } x \in B) \text{ or } x \in C$

$\Leftrightarrow x \in A \cup B \text{ or } x \in C$

$\Leftrightarrow x \in (A \cup B) \cup C$

so by definition, $A \cup (B \cup C) = (A \cup B) \cup C$

The proof of (ii) is left to the reader

5. Distributive Laws

If A,B and C be any three sets, then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

For (i)

for all $x \in A \cup (B \cap C)$

$x \in A$ or $x \in B \cap C$

$x \in A$ or ($x \in B$ and $x \in C$)

($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$x \in A \cup B$ and $x \in A \cup C$

$x \in (A \cup B) \cap (A \cup C)$

so by definition $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

The proof of (ii) is left to the reader

6. Absorption Laws

If A and B be any two sets, then

(i) $A \cup (A \cap B) = A$

(ii) $A \cap (A \cup B) = A$

For (i)

$A \cup (A \cap B) = (A \cup A) \cap (A \cup B)$ by distributive law

$= A \cap (A \cup B) = A (Q\, A \cup A = A : \& A \subseteq A \cup B)$

The proof of (ii) is left to the reader

7. De'Morgan's Laws

If U be the universal set and $A, B \subseteq U$, then

(i) $(A \cup B)^1 = A^1 \cap B^1$

(ii) $(A \cap B)^1 = A^1 \cup B^1$

For (i)

for all $x \in (A \cup B)^1$

$\Leftrightarrow x \notin A \cup B$

$\Leftrightarrow x \notin A$ and $x \notin B$

$\Leftrightarrow x \in A^1$ and $x \in B^1$

$\Leftrightarrow x \in A^1 \cap B^1$

so by definition $(A \cup B) = A^1 \cap B^1$

For (ii)

for all $x \in (A \cap B)^1$

$\Leftrightarrow x \notin A \cap B$

$\Leftrightarrow x \notin A$ or $x \notin B$

$\Leftrightarrow x \in A^1$ or $x \in B^1$

$\Leftrightarrow x \in A^1 \cup B^1$

so by definition $(A \cap B)^1 = A^1 \cup B^1$

8. De'Morgans Laws in terms of difference of sets

If A,B and C be any three sets, then

(i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

For (i)

for all $x \in A - (B \cup C)$

$\Leftrightarrow x \in A$ and $x \in B \cup C$

$\Leftrightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Leftrightarrow (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$

$\Leftrightarrow x \in A - B$ and $x \in A - C$

$\Leftrightarrow x \in (A - B) \cap (A - C)$

so by definition $A - (B \cup C) = (A - B) \cap (A - C)$

The proof of (ii) is left to the reader

1.1.10.Worked out problems :

1.If A and B be any two sets, show that

$A \cup B = A \cap B \Leftrightarrow A = B$

Ans. First suppose that

$A \cup B = A \cap B$

Also $A \subseteq A \cup B = A \cap B \subseteq B$

$\therefore A \subseteq B$

then $B \subseteq A \cup B = A \cap B \subseteq A$

$\therefore B \subseteq A$

since $\therefore A \subseteq B$ and $B \subseteq A$ $\therefore A = B$

convercely, suppose that A = B

then $A \cup B = B \cup B = B$

and $A \cap B = B \cap B = B$

$\therefore A \cup B = A \cap B$

2. If $A \cup X = A \cup Y$ and $A \cap X = A \cap Y$

show that X = Y

Ans. suppose that

$A \cup X = A \cup Y$ and $A \cap X = A \cap Y$

Then for all $x \in$ $\Rightarrow x \in A \cup X$

$\Rightarrow x \in A \cup Y$

$\Rightarrow x \in A$ or $x \in Y$

$\Rightarrow x \in A \cap X$ or $x \in Y (\because x \in X$ and $x \in A)$

$\Rightarrow x \in A \cap Y$ or $x \in Y$

$\Rightarrow (x \in A$ and $x \in Y)$ or $x \in Y$

$\Rightarrow x \in Y$

so $X \subseteq Y$

similarly, we can show that $Y \subseteq X$

This gives X = Y

3. If A and B be any two sets, show that

$A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$

Ans. R.H.S $= \left\{\left(A \cap B^1\right) \cup \left(A \cap B\right)\right\} \cup \left(B \cap A^1\right)$ $\left(\because A - B = A \cap B^1\right)$

$= \left\{A \cap \left(B^1 \cup B\right)\right\} \cup \left(B \cap A^1\right)$ by distributive law

$= (A \cap \Omega) \cup \left(B \cap A^1\right)$ $\left(\because B \cup B^1 = \Omega\right)$

$= A \cup \left(B \cap A^1\right)$ $(\because A \subseteq \Omega)$

$= \quad (A \cup B) \cap \left(A \cup A^1\right)$ by distributive law

$= \quad (A \cup B) \cap \Omega = A \cup B \quad \left(Q\ A \cup B \subseteq \Omega\right)$

4. If A and B be any two sets, show that

(i) $P(A) \cap P(B) = P(A \cap B)$

(ii) $P(A) \cup P(B) \neq P(A \cup B)$

Ans. For (i)

for all $C \in P(A) \cap P(B)$

$\Leftrightarrow C \in P(A)$ and $C \in P(B)$

$\Leftrightarrow C \subseteq A$ and $C \subseteq B$

$\Leftrightarrow C \subseteq A \cap B$

$\Leftrightarrow C \in P(A \cap B)$

so by definition of equality of sets,

$P(A) \cap P(B) = P(A \cap B)$

For (ii) for this , we give the following example

Let A $=\{1\}$ and B$=\{2\}$, then $A \cap B = \{1, 2\}$

Now $P(A) = \{\phi, \{1\}\}, P(B) = \{\phi, \{2\}\}$

$P(A \cup B) = \left\{\phi, \{1\}, \{2\}, \{1, 2\}\right\}$

Also $P(A) \cup P(B) = \left\{\phi, \{1\}, \{2\}\right\}$

so, $P(A) \cup P(B) \neq P(A \cup B)$

5. If A and B be any two sets, show that

(i) $A \Delta B = B \Delta A$

(ii) $A \Delta B = (A \cup B) - (A \cap B)$

Ans. For (i)

$A \Delta B \quad = (A - B) \cup (B - A)$

$= (B - A) \cup (A - B) \quad \left(Q\ A \cup B = B \cup A\right)$

$= \quad B \Delta A$

For (ii) $\quad (A \cup B) - (A \cap B)$

$= (A \cup B) \cap (A \cap B)^1 \left(A - B = A \cap B^1\right)$

$= (A \cup B) \cap \left(A^1 \cup B^1\right)$ by De'Morgan's Law

$= \left\{(A \cup B) \cap A^1\right\} \cup \left\{(A \cup B) \cap B^1\right\}$, by distributive law

$= \left\{\left(A \cap A^1\right) \cup \left(B \cap A^1\right)\right\} \cup \left\{\left(A \cap B^1\right) \cup \left(B \cap B^1\right)\right\}$

$= \left\{\phi \cup \left(B \cap A^1\right)\right\} \cup \left\{\left(A \cap B^1\right) \cup \phi\right\} \left(\because A \cap A^1 = \phi\right)$

$= \left(B \cap A^1\right) \cup \left(A \cap B^1\right)$

$= (B - A) \cup (A - B)$

$= (A - B) \cup (B - A) = A \Delta B$

6.(i) If 60% students in a class offered Biology and 75% students offered Mathematics , how many % of students offered both Biology and Mathematics, if any students have offered either Mathematics or Biology ?

Ans. Let B= set of students offering Biology

M= set of students offering Mathematics

then , as given

n(B)=60, n(M)=75, $n(B \cup M) = 100$

we find $n(B \cap M)$

since $n(B \cup M) = n(B) + n(M) - n(B \cap M)$

$\therefore 100 = 60 + 75 - n(B \cap M)$

i.e. $n(B \cap M) = 60 + 75 - 100 = 35$

$\therefore$ 35% students offered both Biology and Mathematics.

(ii) In a survey of 425 persons, it was found that 115 persons drink Tea, 160 persons drink coffee and 80 persons drink both these two drinks.How many of them drink either tea or coffee ? How many of them drink neither of these two drinks ?

Ans. Let T= set of persons taking tea

C= set of persons taking coffee

Then, as given

n(T)=115, n(C)=160, $n(T \cap C) = 80$ and $n(\Omega) = 425$

$\therefore$ Number of persons taking Tea or coffee

$= n(T \cup C) = n(T) \cup n(C) - n(T \cap C)$

=115+160–80=195

Also, number of persons taking neither tea nor coffee

$= n(\Omega) - n(T \cup C) = 425 - 195 = 230$

7. Of the members of three athelatic teams of school, 21 are in the basketball team,26 are in the hockey team and 29 in the football team,Among these, 14 play hockey and basketball; 15 play hockey and football, 12 play football and basketball and 8 are in all these three teams, How many student are there altogether ?

Ans. Let B=set of students playing Basketball

H=set of students playing Hockey

F=set of students playing Football

Then as given

n(B)=21, n(H)=26, n(F)=29, $n(H \cap B) = 14$

$n(H \cap F) = 15, n(F \cap B) = 12 \text{ and } n(B \cap H \cap F) = 8$

We find $n(B \cup H \cup F)$

$\therefore n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

= 21+26+29–14–15–12+8=43

8. An investigator interviewed 100 students to determine preference for the three drinks : Milk(M). Coffee(C) and Tea(T). The report is

10 had all the three drinks, 20 had M and C, 30 had C and T, 25 had M and T , 12 had M only, 5 had C only and 8 had T only. How many take at least one of these three drinks? How many take none of these three drinks ?

Ans. Let M=set of students taking Milk

C=set of students taking Coffee

T=set of students taking Tea

Then as given

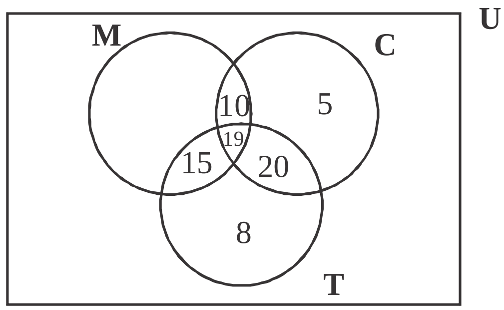
$m(M \cap C \cap T) = 10, n(M \cap C) = 20n, n(C \cap T) = 30, n(M \cap T) = 25$

$n(M) \text{ only} = 12, n(C) \text{only} = 5, n(T) \text{only} = 8, n(\Omega) = 100$

we use Venn diagram method.

First we fill up $n(M \cap C \cap T) = 10$

Next, we compute $n(M \cap C)$.



$n(C \cap T)$ and $n(T \cap M)$ w.r.t $= n(M \cap C \cap T)$

$\therefore n(M \cap C) = 10 + (10) = 20$

$n(C \cap T) = 10 + (20) = 30$

$n(T \cap M) = 10 + (15) = 25$

Finally, we fill up n(M) only = 12, n(C) only = 5 and n(T) only = 8

$\therefore$ Number of students taking at least one of the drink.

$= n(M \cup C \cup T)$

= 12 + 10 + 5 + 15 + 10 + 20 + 8 = 80

Also, numbers persons taking none of these three drinks

$= n(\Omega) - n(M \cup C \cup T) = 100 - 80 = 20$

1.1.11. (Exercise I) (Operation of Sets) :

1. If A and B be any two sets, show that

(i) $(A \cup B) \cap (A \cup B^1) = A$ (ii) $(A \cap B) \cup (A \cap B^1) = A$

2. If A and B be any two sets, prove that

(i) $A \subseteq B \Rightarrow A \cup B = B$ and $A \cap B = A$

(ii) $(A \cap B) \cup (A - B) = A$

3. If A and B be any two sets, sow that

(i) $A \subseteq B \Rightarrow P(A) \subseteq P(B)$

(ii) $P(A) = P(B) \Leftrightarrow A = B$

(iii) $P(A) \cup P(B) \subseteq P(A \cup B)$

Can equality always hold?

4. (i) State and prove Distributive laws for union and intersection of three sets.

   (ii) State and prove De 'Morgan's laws for two sets.

5. Using Venn diagram method, show that

   (i) $A\Delta(B\Delta C)=(A\Delta B)\Delta C$ (ii) $A\cap(B\Delta C)=(A\cap B)\Delta(A\cap C)$

6. In a group of 1000 people, 750 speak Hindi and 400 speak Urdu. How many of them speak both Hindi and Urdu'.

   How many speak Hindi only? How many speak Urdu only?

Ans.150, 600 and 250.

7. A class has 175 student. Following is the description showing number of students studying one or more of the following subjects.

Mathematics 1000, Physics 70, Chemistry 46, Mathematics and Physics 30; Mathematics & Chemistry 28, Physics and Chemistry 23 and all these three subjects 18. How many students are enrolled in Mathematics alone. How many of them are in Physics alone? How many of them are in Chemistry alone ? How many of them take none of these three subjects?

Ans.60,35, 13 and 22.

8. In a survey of 60 people in a locality, it was found that 25 read newspaper H, 26 read newspaper T and 26 read newspaper I ; 9 of them read both H and I; 11 read both H and I, 8 read both T and I ; 3 read all these newspapers. Find the number of people who read at least one of them. Also find the number of people who read exactly one newspapers.

Ans. 52, 30

1.7. Cartesian product of two sets :

1.7.1 Ordered pair

Two numbers a and b listed in a definite order and enclosed by small bracket is called an ordered pair (a, b).

In this ordered pair (a, b), a is called the first coordinats and b is called the second coordinate.

By interchanging the position of the coordinates a and b, the ordered pair changes to (b, a). Thus $(a,b)\neq(b,a)$

Ex. In coordinate Geometry of two dimensions, the position of a point P in xy-plane is determined by an ordered pair (a,b), where a is called x-coordinate and b is called the y-coordinate.

So A(2, 3) and B(3, 2) are two points in xy-plane. Thus $(2,3)\neq(3,2)$

Equality of two ordered pairs.

Let (a,b) (c,d) be two ordered pairs

If (a, b) = (c,d), then we define a = c and b = d

1.7.2 Cartesian product of two sets :

Let A and B be any two non-empty sets. Then the Cartesian product of A and B, written as A × B, is the set of all ordered pairs (x,y) such that $x \in A$ and $y \in B$

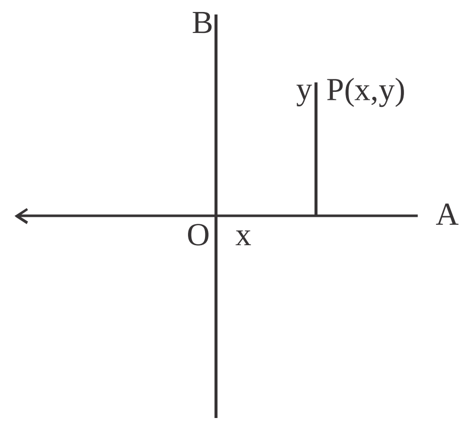
So A × B = Cartesian product of A and B

= {(x, y): $x \in A$ and $y \in B$ }

Since $(x, y) \neq (y, x)$ in general we have

A × B ≠ B × A

Geometrically, If A and B represent x-axis and y-axis respectively in xy-plane, then A × B reprsents the whole xy-plane.



Ex. If A = {1, 3} and B = {2, 4, 5} show that

(i) A × B ≠ B × A (ii) $n(A \times B) = n(B \times A)$ (iii) $n(A \times B) = n(A).n(B)$

Ans. $A \times B = \{1, 3\} \times \{2, 4, 5\}$

$= [\{(1, 2), (1, 4), (1, 5), (3, 2)(3, 4)(3, 5)\}]$

and

$B \times A = \{2, 4, 5\} \times \{1, 3\}$

$= \{(2, 1), (4, 1), (5, 1), (2, 3), (4, 3), (5, 3)\}$

So $A \times B \neq B \times A$

Also $n(A \times B) = 6$ and $n(B \times A) = 6$

So $n(A \times B) = n(B \times A)$

Since $n(A) = 2$ and $n(B)3$ $\therefore$ n(A), n(B) = 2 × 3 = 6

This gives $n(A\times B)=n(A).n(B)=2\times 3=6$

Note – If A and B be any two finite sets then

$n(A\times B)=n(A).n(B)$

1.7.3 Worked out Problems :

1. If A, B, C and D be any four sets such that $A\subseteq C$ and $B\subseteq D$, Prove that $A\times B\subseteq C\times D$.

Ans. Let $A\subseteq C$ and $B\subseteq D$

For all (x, y) $\in A\times B\Rightarrow x\in A$ and $y\in B$

$\Rightarrow x\in A$ and $y\in B\subseteq D$

$\Rightarrow x\in C$ and $y\in D$

$\Rightarrow (x,y)\in C\times D$

So by definition, $A\times B\subseteq C\times D$

2. If A, B and C be any three sets, prove that

(i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

(ii) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

Ans. For (i) For all (x,y) $\in A\times(B\cup C)$

$\Leftrightarrow x\in A$ and $y\in B\cup C$

$\Leftrightarrow x\in A$ and $(y\in B$ or $Y\in C)$

$\Leftrightarrow (x\in A$ and $y\in B)$ or $(x\in A$ or $y\in C)$

$\Leftrightarrow (x,y)\in A\times B$ or $(x,y)\in A\times C$

$\Leftrightarrow (x,y)\in(A\times B)\cup(A\times C)$

So, by definition, $A\times(B\cup C)=(A\times B)\cup(A\times C)$

For (ii)

For all (x, y) $\in A\times(B\cap C)$

$\Leftrightarrow x\in A$ and $y\in B\cap C$

$\Leftrightarrow x\in A$ and $(y\in B$ and $y\in C)$

$\Leftrightarrow (x\in A$ and $y\in B)$ and $(x\in A$ and $y\in C)$

$\Leftrightarrow (x,y)\in A\times B$ and $(x,y)\in A\times C$

$\Leftrightarrow (x,y)\in(A\times B)\cap(A\times C)$