M C A

PART - I

SUBJECT
DISCRETE MATHEMATICS

PAPER - III

PREPARED BY
DR. AMARESH RANJAN
Assistant Professor
Nalanda Open University
Patna
Contact No. : 9279177009

SET, RELATIONS AND FUNCTIONS

Objective

Introduction of Set

Representation of Sets

Types of Set

Set Operation

The Objective

After studying this unit, you should be able to

* Explain what a set, a relation or a function is
* Give Examples and non-examples of sets, relations and functions.
* Perform different operations on sets.
* Establish relationships between oprations on sets and those on statements in logic
* Use venn diagrams
* Explain the difference between a relation and a function
* Describe different types of relations and functions
* Define and Perform the four basic operations on functions.

The concept of set plays a fundamental role in the development of discrete structure. The word 'set' is synonymous with collection. A set is a collections of well defined and distinct object, but not mathematically. After going through this unit, you will be Introduced to the concept of sets, types of sets, operation on sets and the properties common to logic and sets. Further after goin through this unit you will be able to discuss relations and its types, the Properties of relations and cartesian product

Also you will be able to explain function and its types and operation on functions.

Introduction

The concept of sets is fundamental to all branches of mathematics, (The creation of set theory is due to the German Mathematician George Cantor. He was born on march 3rd, 1815 at petersburg in Russia) Consider any dictionary of English. It is a collection of words and their meanings. A word either belongs to this collection or not, depending on whether it is listed in the dictionary or not. This collection is an Example of a set.

The study of sets includes the study of operations on sets. In this unit we discuss the operations of complementation union, Intersection and cartesian Product. We also introduce venn Diagrams, a Pictorial way of describing sets.

Since the material covered in this unit is going to be basis for the rest of the course, Please study it carefully.

Representation of Sets

There are three ways of representing sets. Tabular form, set Builder form and the pictorial representation through venn Diagrams.

4.2.1 Tabular Form or Roster Method or listing method or Enumeration method : In this method all elements of sets are listed with in the brackets { }, each separated from the other by a comma, as in the examples.

The accepted convention for writing a set by the listing method is that elements will not be repeated. For example in the set A = {4, 2, 8, 2, 6}, 2 is repeated, which is not necessary. So will write A = {4, 2, 8, 6}

Repeatation of elements are not allowed in a set.

Set Builder form or Property Method or Rule Method or Algebraic form :

In this method all elements are not written within the brackets but its conditions or properties are written within the brackets.

Examples

N = {x : x is natural number}

I = {x : x is Integers}

A = {x | x is an vowel of English alphabet}

B = {x | x is a natural number < 5}

SET :

Set is a collection of well defined and distinct object. Set is denoted by capital letters of English alphabet.

Example of Sets :

(i) V = {a, e, i, o, u}

(ii) The set of all students of NOU Patna

(iii) The set of even Integers 2, 4, 6, 8

(iv) The set of all natural numbers.

Examples of not a set :

(i) Best Cricketer of India

(ii) Intelligent students of the class

(iii) Honest persons of this state

(iv) Beautiful girls of India.

The members of a set are called elements. We always use { } brackets to denote a set.

Types of Sets

Finite Set : If the number of elements of a set is finite, the set is said to be a finite set.

Examples

(i) The set of all days of a week

(ii) The set of all the months of a year

(iii) The set of all the Assistant Professors in PU, Patna

Infinite Set :

In this set th number of elements are not countable, the set is said to be infinite set.

Example :

(i) Let E be the set of all positive even Integers.

i.e. E = {x : x is a positive even integers)

(ii) The set of all the straight lines perperndicular to the x–axis is infinite

Singleton set or Unit Set :

A set having only one element is called a Singleton set. or Unit Set

Example :

(i) Present Prime Minister of India

(ii) Present Chief Minister of Bihar

Null set or Empty Set or Void set = { }

A Set consisting of no element is called a null set and is denoted by or { }

Example :

A = {x : x is an even number which is not divisible by 2}

B = {Set of immortalmen}

C = {Set of all married Bachelor}

Note : {0} is not a null set becaulse it contains one element O. So the set {0} is a singleton Set.

4.3.5 Set of Sets :

If a set is an element of another set, then the latter set is called the set of sets.

Examples :

A = {2, 3, 4, (5, 6), 7}

2 A	4 A	6 A	7 A
3 A	5 A	(5,6) A	

So (5, 6) is set which is an element of set A.

4.3.6 Subsets :

If A and B are two sets and if every element of set A is an element of set B, We say that A is a subset of B.

Symbolically it is written as

A _ B Ex. A ={1, 2, 3}, B = {1, 2, 3, 4, 5} i.e. A B

i.e. null set is subset of every set.

Equality of Sets :

Two sets are said to be equal if and ony if they have the same elements.

Examples :

A = {2, 4, 6}, B = {6, 4, 2}

Thus Set A and Set B are two equal sets.

Proper subsets :

If A and B are two non-empty sets and every elements of a set A is an element of set B and set B contains at least one element which does not belong to set A then set A is called proper sub set of B and Set B is called super set of set A.

Examples :

A = {1, 2, 3}

B = {1, 2, 3, 4, 5}

A B

B A

Equaivalent Set

In this set the number of elements are equal.

Example :

A = {1, 2, 3} So A B

B = {a, b, c}

4.3.10 Power Set :

The set formed by all the subset of a given set is called the power set of the given set.

Total number of elements in the power set is $(2)^n$. Where n = Number of elements.

Example :

A = {2, 3, 4}

P(A) ={ ,(2),(3),(4),(2,3), (2, 4), (3, 4), (2, 3, 4)}

Total number of elements in the Power set is $(2)^3$ ie $2 \times 2 \times 2 = 8$

Universal Set = U or S or

If all the sets under investigation are subsets of the fixed set then this fixed set is called the universal set.

Examples :

U= S={1, 2, 3, ...10}

A = {2, 4, 6, 8, 10}

B = {1, 3, 5, 7, 9}

Here, U is a universal set and set A and Set B are sub sets of the fixed set(U).

Cardinal number of a finite set :

It is the number of distinct elements of the Set A and this number is denoted by n(A).

Examples :

A = {1, 2, 2, 3, 3, 3, 4}

Cardinal number of set is 4

n(A) = 4

because repeatations are not allowed in a set.

SET OPERATION

Union of two Sets :

The union (or sum or logical sum or join) of the sets A and B is the set of all points which belong either to set A or to Set B or to both.

Symbolically it is written as

A B={x A or x B or x Both}

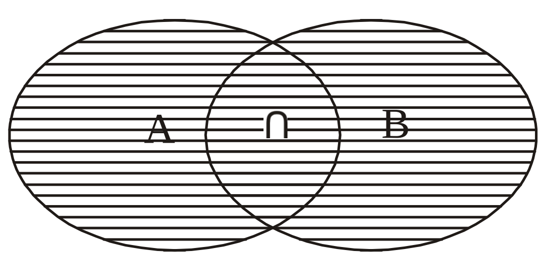
Examples :

A = {1, 2, 3, 4}

B = {2, 4, 6, 8}

A B={1, 2, 3, 4, 6, 8}

With the help of venn Diagram it is presented as



A∩B

Intersection of Two sets :

The inter section or (product or meet) of the sets A and B is the set of all points which belong to both A and B.

Symbolically it is written as

A B={x:x A and x B}

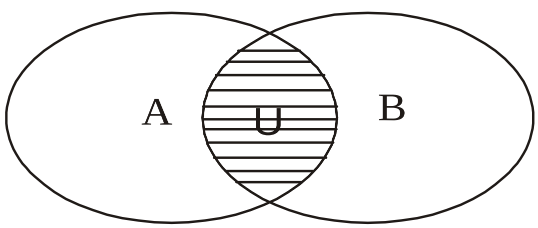
Examples :

A = {1, 2, 3, 4}

B = {2, 4, 6, 9}

A B={2, 4}

with the help of venn Diagram



AUB

Disjoint Sets :

The two sets are said to be disjoint if they have no common element.

The sets A and B are disjoint if no element of A is in B and no element of B is in A.

Hence A and B are disjoint if $A \cap B=$

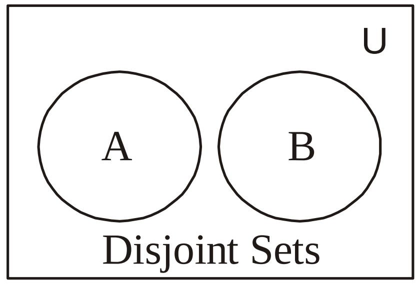
Examples :

If A = {1, 3, 5, 7}

B = {2, 4, 6, 8}

are said to be disjoint sets because they have no common element.

With the help of venn Diagram



Difference Operation :

If A and B are sets, the difference between A and B is the set

A – B defined by

$$A–B = \{x:x \quad A \text{ and } x \quad B\}$$

i.e. A–B is the set of all those elements of A which donot belong to B.

Similarly,

$$B–A=\{x:x \quad B \text{ and } x \quad A\}$$

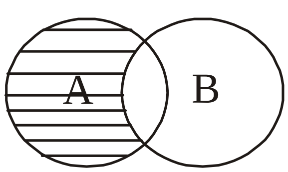
Examples :

A = {2, 5, 8, 11} and B= {5, 11, 17}

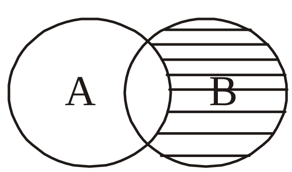
A – B = {2, 8}

B – A = {17}

with the help of venn Diagram



A – B



B – A

Complement of a set or (Absolute complement) :

Let A be a subset of the unversal set U. The set of all those elements of the universal set $\bigcup$, which are not belonging to the given set A, is called the complement of set A.

The complement of set A is denoted by A´ or A^c or $\overline{A}$

Symbolically it is written as

$$A'=\{x:x \quad U \text{ and } x \quad A\}$$

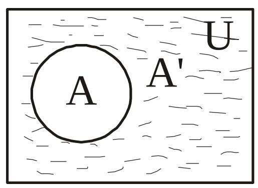
Examples :

U = {1, 2, 3, 4, ...10}

A = {1, 3, 5, 7, 9}

$$A'=\overline{A} \quad U-A=\{2, 4, 6, 8, 10\}$$

<u>WITH THE HELP OF VENN DIAGRAM</u>



Symmetric Difference of Two Sets

Let A and B be any two sets then the set (A–B) U(B–A) is called the symmetric difference of A and B.

It is denoted by $A \quad B$.

$$A \quad B=(A-B)\cup(B-A)$$

Symbolically,

$$A \quad B=\{x:x \quad A \text{ and } x \quad B \text{ or } x \quad B \text{ and } x \quad A\}$$

Examples :

A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7}

A–B = {1, 2, 3}

B–A = {6, 7}

A B= (A–B) $\bigcup$ (B–A)

={1, 2, 3) $\bigcup$ {6, 7}

A B= {1, 2, 3, 6, 7}

With the help of Venn Diagram