M C A

PART - I

SUBJECT
DISCRETE MATHEMATICS

PAPER - III

PREPARED BY
DR. AMARESH RANJAN
Assistant Professor
Nalanda Open University
Patna
Contact No. : 9279177009

SET, RELATIONS AND FUNCTIONS

Types of Relation

Partial order Relation

Summary

Questions for Exercise

Types of Relations

Reflexive Relation :

A relation R on a set A is said to be reflexive if and only if aRa, for all a A that is, (a,a) R for all a A

Example :

If A = a set of triangles in a plane and aRb be defined as "triangle a is similar to triangle b", then R is reflexive, because every triangle is similar to itself, that is, a Ra is true in this case.

Symmetric Relation :

A relation R on a set A is said to be symmetric iff (i.e if and only if) aRb bRa for all a,b A that is

(a,b) R (b, a) R for all a,b A

Example :

If A = a set of triangles in a plane and aRb be defined as "triangle a is similar to triangle b" then R is symmetric because if triangle a is similar to triangle b, then triangle b is also similar to triangle a, that is, aRb bRa for all a, b A .

Transitive Relation :

A relation R on a set A is said to be transitive iff aRb and bRc aRc for all a, b, c, A that is, (a, b) R and (b, c) R (a, c) R for all a, b, c A .

Example :

If A = a set of all straight lines in a plane and aRb be defined as "The line a is parallel to the line b" Then R is transitive, because if, the line a is parallel to the line b and the line b is parallel to the line c, then the line a is parallel to the line C. That is

In this case, aR a = bb and bRc aRc for all a, b, c A.

Anti symmetric Relations :

A relation R on a set A is said to be anti-symmetric iff aRb and bRa a = b for all a, b A that is (a, b) R and (b, a) R a = b for a, b A .

Example : let N = The set of all natural numbers.

Let a relation R on A be defined by “a is a divisor of b”, ∀ a, b N Clearly R is anti symmetric because if a divides b and b divides a then a = b.

Equivalence Relations and partitions :

In Mathematics, an equivalence relations is the relation that holds between two elements if and only if they are members of the same cells within a set that has been partitioned into cells such that every element of the set is a member of one and only one cell of the partition. The Intersection of any two different cells is empty and the union of all the cells equals the original set.

Mathematically, these cells are called equivalence classes. A given binary relation on a set X is said to be equivalence relation if and only if it is reflexive, symmetric and transitive.

Example : Let N be the set of natural number. Define R on N as

R={(x,y) : x+y is even, x, y N}

Proof : Let x N . Now $x + x = 2x$

Clearly $2x$ is even. Therefore R is reflexive let x, y N and x + y is even Clearly y + x is also even and hence R is symmetric.

Now if x + y is even and y + z are even then we have to prove that x + z is even. Since x + y and y + z are even both (x + y) and (y + z) are divisible by 2.

(x + y) + (y + z) is also divisible by 2 *i.e.* x + (y + y) + z is divisible by 2

(x + z) is divisible by 2.

Hence R is transitive. So, R is an equivalence relation.

Prove that any two equivalence classes are Identical or disjoint.

Solutions : First we shall prove that (a, b)   R.

This Implies that $[a]_R = [b]_R$

Suppose (a, b)   R.

Case I [a] = [b]

Let $x \quad [a] \quad (x,a) \quad$ R

$(x,\text{b}) \quad$ R $[\because (x,a) \quad$ R and $(a,b) \quad$ R and R is transitive]

$x \quad$ b

$[a] \quad [\text{b}]$

Now suppose [a], [b] are two equivalence classes.

Case II |a| = |b| or |a| $\bigcap$ |b| =

If $[a] \bigcap [\text{b}] =$ then nothing to prove

Suppose $[a] \bigcap [\text{b}] =$ then $x \quad [a] \bigcap [b]$

$x \quad [a]$ and $x \quad [b]$

$(x,a) \quad$ R and $(x,b) \quad$ R

[x] = [a] and [x] = [b]

[a] = [b]

$[a] \bigcap [b]$ or [a] = [b]

i.e. any two equivalence classes are identical or disjoint. Hence Prove.

4.9.7 If the function $f : \mathrm{R} \quad \mathrm{R}$ be defined by $f\,|x| \quad x^2 \quad 1$ where R is the set of real numbers then find the value of

(i) $f^{-1}(-5)$ (ii) $f^{-1}(26)$ (iii) $f^{-1}(10, 37)$

Solution :

(i) $f^{-1}(-5) = \{x \quad \mathrm{R}: f(x) \quad -5\}$

$\{x \quad \mathrm{R}: x^2 \quad 1 \quad -5\}$

$\{x \quad \mathrm{R}: x^2 \quad -6\}$

$\{x \quad \mathrm{R}: x \quad \sqrt{6}\,i$

$(as \quad \sqrt{6}i$ is imaginary number.

(ii) $f^{-1}(26) \quad \{x \quad \mathrm{R}: f(x) \quad 26\}$

$\{x \quad \mathrm{R}: x^2 \quad 1 \quad 26\}$

$\{x \quad \mathrm{R}: x^2 \quad 25\}$

$\{x \quad \mathrm{R}: x \quad 5\}$, $=\{5, -5\}$

(iii) $f^{-1}(10, 37) \quad \{x \quad \mathrm{R}: f(x) \quad 10, f(x) \quad 37\}$

$\{x \quad \mathrm{R}: x^2 \quad 1 \quad 10, x^2 \quad 1 \quad 37\}$

$\{x \quad \mathrm{R}: x^2 \quad 9, x^2 \quad 36\}$

$\{x \quad \mathrm{R}: x \quad 3, x \quad 6\}$

$= \{3, -3, 6, -6\}$

4.9.8 Questions : In a set S = {a, b, c, d} of four men a is younger to the other three b is younger to c and d only and c is younger to d only.

Is the relation "is younger to" an equivalence relation ? Give reason for your answer.

Solutions : It is not an equivalence relation.

If b is younger to c, then c is not younger to b. Hence it is not an equivalence relation.

Question :

Consider the non-empty set consisting of children in a family, state giving reasons, whether each of the following is

(i) Symmetric (ii) transitive

(a) x is a brother of "y   (b) x "likes" y.

Solution (a) x is a brother of y does not necessarily imply that y is a brother of x, because y may be the sister of x.

Hence the relation "is a brother of" is not symmetric.

x is a brother of y and y is a brother of z clearly imply that x is a brother of Z.

Hence the relation, "is a brother of" is transitive.

(b) If x likes y, then it is not necessary that y may like x.

Hence the relation "likes "is not symmetric.

If x likes y and y likes Z, then it is not necessary that x may like Z.

Hence the relation "like" is not transitive.

Questions :

Let N × N be the set of ordered pairs of natural numbers. Let R be the relation in N × N defined by (a, b) R(c, d) if and only if ad = bc then Prove that R is an equivalence relation.

Solutions : N×N = {(a, b); a, b N}, N being the set of natural numbers.

(i) R will be reflexive if $\forall$(a, b) N N (a, b) R (a, b) that is if ab = ba which is true.

(ii) R will be symmetric if (a, b) R(c,d) (c, d) R(a, b) that is if ad = bc

cb = da

That is if ad = bc bc = ad

∵ cb = bc and da = ad due to reflexive property) which is true.

(iii) R will be transitive if

(a, b) R (c, d), (c, d) R(e, g) (a, b) R (e, f)

That is if ad = bc, cf = de af = be.

That is if (on multiplying) adcf = bdce

af = be

That is if af = be af =be

which is true

Since R is reflexive, Symmetric and transitive therefore R is an equaivalence relation on N×N

Partial order Relation :

A relation R in a set A is said to be a partial order relation if it is

(i) reflexive that is, a Ra is true , $\forall$ a A

(ii) Anti symmetric, that is, if a Rb and bRa

a = b, $\forall$ a, b A and

(iii) transitive that is aRb and bRc

aRc, $\forall$ a, b, c A .

for Example

Let there be a relation on R defined by $\leq$

Then a $\leq$ b where the sign < has its usual meaning and a, b, c R be arbitrary.

Then the relation is reflexive as a = a, anti symmetric as a $\leq$ b, b $\leq$ a a = b

and transitive as

a $\leq$ b, b $\leq$ c a $\leq$ c

Hence the relation $\leq$ under consideration is a partial order relation on R.

Partially Ordered Set :

Let the partial order relation in the set A be $\leq$

Then the pair (A, $\leq$) is called a partially ordered set.

Bounds of an ordered set :

Let a partially ordered set be defined by (A, $\leq$). Let B (A, $\leq$) and a A. If a $\leq$ x, $\forall x$ B, then a is said to be the lower bound of B. If x $\leq$ b, $\forall x$ B then b A is called the upper bound of B.

The smallest member of the collection of all upper bounds of B. if it is exists, is called the least upper bound or the supremum of B.

The greatest member of the collection of all lower bounds of B, if it exists, is called the greatest lower bound or Infimum of B.

4.10.4 Lattice

A lattice is a partially ordered set (L, ≤) Provided each pair (x, y) of L has a greatest lower bound and a least upper bound in L.

Given an example of a partially ordered set which is not a lattice.

Solutions :

Let X = {1, 2, 3, 4, 5, 6, 7, 8}

let the partial ordered set be (x, *l*) where *l* is defined by

'x divides y' and x, y x.

Here 4 v 6 = least upper bound = L.C.M. of 4 and 6 = $12 / x$

Hence (x, l) is not a lattice.

Summary

(i) Sets are one of the most fundamental concepts in mathematics. A set is a collection of distinct object considered as a whole. The concept of set plays a fundamental role in the development of discrete structure. The word 'set' is synonymous with collection. The memebers of a set are called elements. We always use { } brackets to denot a set. A set which has finite number of elements is called a finite set, else it is called an infinite set.

(ii) The three operations of sets namely union, Intersection and complement are analogous to the operations of addition, multiplication and subtraction of numbers, respectively. The union of any two sets A and B is the set of all those elements x such that x belongs to at least one of the two sets A nd B. The intersection of two sets, A and B, is the set of all those elements x, such that x belongs to both A and B and is denoted by $A \cap B$. If A and B are two sets, then complements of B relative to A is the set of all those elements x A such that x / B and is denoted by A – B.

(iii) In Mathematics, an equivalence relation is the relation that holds between two elements if and only if they are members of the same cell within a set that has been partitioned into cells such that every element of the set is a member of one and only one cells of the partition. The intersection of any two different cells is empty and the union of all the cells equals the original set. Mathematically these cells are called equivalence classes. A relation

R on a set A is called an equivalence relation if R is reflexive, symmetric and transitive. In mathematics and Specifically in graph theroy, a digraph or directed graph is a graph or set of nodes that are connected by edges, where the edges have a direction associated with them.

(iv) A function or mapping from a set A to Set B is a method' which pairs elements of the set A with unique elements of the set B and we denot f : A B to indicate that f is a function from the set A to the set B. The inverse function of f is the function that assigns to an element b B the unique element a in A such that f(a) = b. The inverse function of f is denoted by f^{-1}. A function f is called Primitive recursive if it can be obtained from the initial function by a finite number of operations of compasition and recursion. A function is said to be partial recursive if it can be obtained from the initial functions by a finite numbers of applications of the operations of composition, recursion and minimization.

Questions for Exercise :

(i) Define the set and its types

(ii) Define union and intersection of set

(iii) Define power set and universal sets.

(iv) Define the function and different types of function

(v) If U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {0, 1, 2, 3, 4}, B = {1, 2, 3}

C = {5, 6, 7}, D = {5, 7, 8, 9}

Find the following :

(a) AUB (b) CUD (c) DUB (d) A – B (e) C – D (f) A – D.