B.C.A -PART - I

SUBJECT : MATHEMATICS
PAPER - IV

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ARITHMETIC PROGRESSION

4.1 SEQUENCE :

An ordered collection of numbers t_1, t_2, t_3, t_4....t_n....is a sequence if according to some definite rule or law, there is definite value of tn called the term or element of the sequence corresponding to any value of the natural number n.

t_1 = First term

t_2 = Second term

t_3 = Third term

t_n = nth term

Example 2, 4, 6, 8, 10

First = t_1 = 2

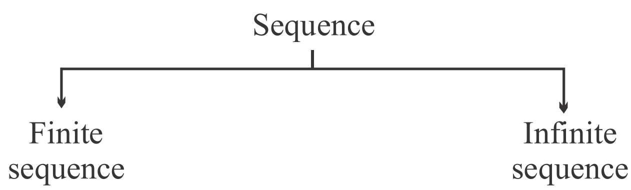
Second term = t_2 = 4

Third term = t_3 = 6

Fourth term = t_4 = 8

Fifth term = t_5 = 10

Sequence is of two types



(a) Finite sequence is called finite if the number of terms is countable In this case there is a last term.

Ex. 1, 3, 5, 7, 9

Here first term = 1

last term = 9

(b) Infinite sequence : In this sequence the number of terms are not countable. In this case there is no last term.

Ex 2, 4, 6, 8.....

4.1. SERIES :

When we introduce +or –sign among the terms of sequence then the sequence is known as series.

Ex.10 + 20 + 30 + 40 + 50

2 + 0 – 2 – 4 – 6 – 8

Series is also of two types

(a) Finite series

(b) Infinite series

4.2. ARITHMETIC PROGRESSION (A.P) :

The series in which each term increases or decreases by a common number continuously is called Arithmetic progression.

i.e. It is difference between term and its preceding term is a constant and this constant is known as coomon difference between difference. Common difference is donoted by d

Ex. 15, 30, 45, 60, 75

$30 - 15 = 15 = 45 - 30$

$t_1, t_2, t_3, t_4, ...tn$

A/c to definition of an AP

4.3. GENERAL TERM OR nth TERM OF AN AP:

Proof : Let a be the first term, d be the common difference, n be the number of term.In be the nth term of an AP.

$t_1 = a = a + (1 - 1)d$

A/c to definition of an AP

$t_2 - t_1 = d = t_3 - t_2 = t_4 - t_3$

$t_2 - t_1 = d \Rightarrow t_2 = t_1 + d$

$$t_2 = a + d = a + (2-1)d$$

$t_3 - t_2 = d \Rightarrow t_3 = t_2 + d = a + d + d$

$$t_3 = a + 2d = a + (3-1)d$$

$t_4 - t_3 = d \Rightarrow t_4 = t_3 + d = a + 2d + d$

$$t_4 = a + 3d = a + (4-1)d$$

Similarly

$tn = a + (n-1)d$

4.4. PROPERTTES OF ARITHMATIC PROGRESSION :

(a) If the same quantity be added to or subtracted from, all the terms of an arithmetic series, the resulting series is also an arithmetic series.

(b) If all the terms of an arithmetic series be multiplied or divided by the same quantity, the resulting series is also an arithmetic series.

(c) If the corresponding terms of two arithmetic series be added or subtracted the resulting series, is also an artihmetic series.

4.6. PROOF :

To find the sum of n terms of an AP.

Let a be first terms and d be the common difference, n be the number of terms, l be the last termsn be the sum to n terms of an AP.

$$S_n = a + (a+d) + (a+2d) + ... + (l-d) + l$$

Now Equation (i) is writing in reverse order

$$S_n = l + (l-d) + (l-2d) + ... + (a+d) + a$$

Adding Equation (i) and (ii) we get

$$S_n = a + (a+d) + (a+2d) + ... + (l-d) + l$$

$$S_n = l + (l-d) + (l-2d) + ... + (a+d) \neq a$$

$$2S_n = (a+l) + (a+l) + (a+l) + ... + (a+l) + (a+l)$$
$$2S_n = n(a+l)$$

$$\boxed{S_n = \frac{n(a+l)}{2}}$$

Where $l = t_n = a + (n-1)d$

$$S_n = \frac{n}{2}\{a + a + (n-1)d\}$$

$$\boxed{S_n = \frac{n}{2}\{2a + (n-1)d\}}$$

4.6. ARITHMETIC MEAN (M) :

(a) Single AM : If three quantities be in AP then middle one is single AM between them.

Let a and b be the two quantities and x be the single AM between them.

a, x, b one in AP

$x - a = b - x$

$x + x = a + b$

$2a = a + b$

$\boxed{x = \frac{a+b}{2}}$ Single AM

(b) To insert n AMs between two given quantities.

Proof : Let a and b be the two given quantities and $x_1, x_2, x_3.....x_n$ be the n AMs between them.

$\because$ a, $x_1, x_2, x_3.....x_n$, b are in AP

First term = a

last term = b

Total number of terms = n + 2

Let d be the common difference

$$l = t_n = a + (n-1)d$$

$$b = a + (n+2-1)d$$

$$b - a = (n+1)d$$

$$\boxed{\frac{b-a}{n+1} = d}$$

$$x_1 = \text{2nd term} = \text{1st AM} = a + d$$

$$x_1 = a + \frac{b-a}{n+1}$$

$$x_2 = \text{3rd term} = \text{2nd AM} = a + 2d$$

$$x_2 = a + \frac{2(b-a)}{n+1}$$

$$x_3 = \text{fourth term} = \text{3rd AM} = a + 3d$$

$$x_3 = a + \frac{3(b-a)}{n+1}$$

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$x_n = (n+1)$ th term = nth AM = a + nd

$$x_n = a + \frac{n(b-a)}{n+1}$$

4.7. ILLUSTRATION :

4.7.1 Find 17th term of the series AP.

20, 18, 16, 14

Sol: Here first term = a = 20

d = 18 – 20 = –2, n = 17

$$t_n = a + (n-1)d$$

$$t_{17} = 20 + (17-1)(-2)$$

$$t_{17} = 20 + 16(-2) = 20 - 32 = -12$$

4.7.2. Find the first term and the common difference of the progression where 8th and 102nd terms are respectively 23 and 305.

Soln. Let a be the first term, d be the common difference.

$$t_n = a + (n-1)d$$

$$t_8 = a + (8-1)d = a + 7d = 23$$

$$t_{102} = a + (102-1)d = a + 101d = 305$$

Eqn. (ii) – Eqn (i)

a + 101 d = 305

a + 7d = 23

94d = 282

$$d = \frac{282}{94}$$

d = 3

From (i)

a + 7d = 23

a + 7 × 3 = 23

a = 23 – 21 = 2

First term = a = 2

Commn difference = d = 3

4.7.3 Find the middle term of the AP 5, 8, 11, ...65

Soln. $a = 5$, $d = 8 - 5 = 3$, $l = t_n = 65$

$$l = t_n = a + (n-1)d$$

$$65 = 5 + (n-1)3$$

$$65 - 5 = (n-1)3$$

$$\frac{60}{3} = n - 1$$

$20 = n - 1$

$n = 21$

$$\text{middle term} = \frac{n+1}{2} = \frac{21+1}{2} = 11\text{th term}$$

$$t_n = a + (n-1)d$$

$$t_{11} = 5 + (11-1)3 = 5 + 10 \times 3$$

$$t_{11} = 35$$

4.7.4. The pth, the qth and the rth term of an AP are respectively x, y and z then prove that

$$x(q-r) + y(r-p) + z(p-q) = 0$$

Sol. Let a = the lst term, d = common difference A/c to Question

$x = a + (p - 1)d$ ...(i)

$y = a + (q - 1)d$ ...(ii)

$z = z + (r - 1)d$ ....(iii)

LHS = $x(q-r)+y(r-p) +z(p - q)$

$$= \{a + (p-1)d\}(q-r) + \{a + (q-1)d\}(r-p) + \{a + (r-1)d\}(p-q)$$

$$= a\{q - r + r - p + p - q\} + d\{(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)\}$$

$$= a \times 0 + d + |pq - pr - q + r + qr - pq - r + p + pr - qr - p + q|$$

$= a \times 0 + d \times 0 = 0 =$ RHS.

4.7.5. Find the sum of the series $1\frac{1}{7}, 1, \frac{6}{7}$ upto 16 terms

Soln. $a = 1\frac{1}{7} = 8/7, d = 1 - \frac{8}{7} = d\frac{7-8}{7} = -\frac{1}{7}$

n = 16

We know that

$$S_n = \frac{n}{2}\{.2a + (n-1)d.\}$$

$$S_{16} = \frac{16}{2}\left(2 \times \frac{8}{7} + (16-1)\left(-\frac{1}{7}\right)\right)$$

$$S_{16} = 8\left(\frac{16}{7} - \frac{15}{7}\right) = \frac{8 \times 1}{7} = 8/7$$

4.7.6. Find the sum of odd numbers from 5 to 100.

Soln. S = 5 + 7 + 9 + ...+ 99

a = 5, d = 7 – 5 = 2, l = 99

$l = t_n = a + (n-1)\,d$

99 = 5 + (n –1)2

99 – 5 = (n –1)2

$$\frac{94}{2} = n - 1$$

47 = n – 1

n = 48

$$S_n = \frac{n}{2}(a + l) = \frac{48}{2}(5 + 99)$$

$$S_{48} = 24 \times 104 = 2496$$

4.7.7 The sum of the n terms of an AP whose first term is 5 and common difference is 36, is equal to the sum of 2n terms of another AP. Whose first term is 36 and common difference is 5. Find n.

Soln. Sum of n terms of an AP

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_1 \Rightarrow n_1 = n\ a_1 = 5, d_1 = 36$$

$$S_2 \Rightarrow n_2 = 2n, a_2 = 36, d_2 = 5$$

$$S_1 = \frac{n_1}{2}\left[2a_1 + (n_1 - 1)d_1\right]$$

$$S_1 = \frac{n}{2}\left[2 \times 5 + (n-1)36\right]$$

$$S_1 = \frac{n}{2}(10 + 36n - 36) = \frac{n}{2}(36n - 26)$$

$$S_1 = \frac{n}{2} \times 2(18n - 13)$$

$$S_1 = n(18n - 13)$$

$$S_2 = \frac{n_2}{2}\left\{2a_2 + (n_2 - 1)d_2\right\}$$

$$S_2 = \frac{2n}{2}\left(2 \times 36 + (2n-1)5\right)$$

$$S_2 = n(72 + 10n - 5) = n(67 + 10n)$$

A/c to Question

$$S_1 = S_2$$

$$n(18n - 13) = n(67 + 10n)$$

$$\Rightarrow 18n - 13 = 67 + 10n$$

$$\Rightarrow 18n - 10n = 67 + 13$$

8n = 80

$$\boxed{n = 10}$$

4.7.8. How many terms of the progression 15, 12, 9....should be taken so that their sum is 36? Interprete the double answer.

Soln. $a = 15, d = 12 - 15 = -3, S_n = 36$

$$S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$$

$$36 = \frac{n}{2}\left\{2 \times 15 + (n-1)(-3)\right\}$$

$$36 = \frac{n}{2}(30 - 3n + 3)$$

$\frac{36}{1} = \frac{n}{2}(33 - 3n)$

72 = 3n (11 – n)

$\frac{72}{3} n(11 - n)$

$24 = 11n - n^2$

$\Rightarrow n^2 - 11n + 24 = 0$

$\Rightarrow n^2 - 8n - 3n + 24 = 0$

$\Rightarrow n(n-8) - 3(n-8) = 0$

$\Rightarrow (n-3)(n-8) = 0$

$n - 3 = 0 \qquad n - 8 = 0$

$\boxed{n = 3} \qquad \boxed{n = 8}$

Sum of first three term is 36

i.e. $S_3 = 15 + 12 + 9 = 36$

Sum of first eight term is 36

i.e. $S_8 = 15 + 12 + 9 + 6 + 3 + 0 - 3 - 6 = 36$

4.7.9 Tenth term of an AP is 10 more than fifth term and their sum is 32. Find the series.

Soln. $t_{10} = 10 + t_5$ (i) and

$t_5 + t_{10} = 32$ (ii)

We know that

$t_n = a + (n-1)d$

$t_5 = a + (5-1)d = a + 4d$

$t_{10} = a + (10-1)d = a + 9d$

From (i)

$t_{10} = 10 + t_5$

$\Rightarrow a + 9d = 10 + a + 4d$

$\Rightarrow 9d - 4d = 10$

$\Rightarrow 5d = 10$

$\boxed{d = 2}$

From (ii)

$t_5 + t_{10} = 32$

$\Rightarrow a + 4d + a + 9d = 32$

$\Rightarrow 2a + 13d = 32$

$\Rightarrow 2a + 13 \times 2 = 32$

$\Rightarrow 2a = 32 - 26$

$\Rightarrow \frac{2a = 6}{|a = 3|}$

The series of an AP

a, a + d, a + 2d, a + 3d

3, 5, 7, 9, 11,......

4.7.10.Find common difference of an AP if first term is 1, 1ast term is 50 and the sum of the series is 204.

Soln. $a = 1, l = 50, S_n = 204$

$$S_n = \frac{n}{2}(a + l)$$

$$204 = \frac{n}{2}(1 + 50)$$

$$2 \times 204 = 51n$$

$$n = \frac{2 \times 204}{51}$$

$\boxed{n = 8}$

$$l = t_n a + (n-1)d$$

$$50 = 1 + (8-1)d$$

$$50 - 1 = 7d$$

$$49 = 7d$$

$$d = 7$$