B.C.A -PART - I

SUBJECT : MATHEMATICS
PAPER - IV

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GEOMETRIC PROGRESSION (G.P)

4.2 GEOMETRIC PROGRESSION (G.P.):

In this sequence, in which the ratio of any terms and the term preceding it is always a constant quantity, is called a geometric progression.

i.e. It is ratio of term and its preceding term is a constant and this constant is known as common ratio. Common ratio is denoted by r.

Ex. 2, 4, 8, 16, 32, 64

$$\frac{4}{2} = \boxed{2} = \frac{8}{4} = \frac{16}{4} = \frac{32}{16}$$

$$t_1, t_2, t_3, t_4 ...... t_n$$

A/c to definition of a GP

$$\frac{t_2}{t_1} = \boxed{r} = \frac{t_3}{t_2} = \frac{t_4}{t_3} =$$

4.2.1.GENERAL TERM OR nth TERM OF A GP:

Proof : Let a be the first term, r be the common ratio, n be the number of terms, tn be the nth term of a GP.

$$t_1 = a = a \times (r)^{(1-1)}$$

A/c to definition of a GP

$$\frac{t_2}{t_1} = \boxed{r} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{t_5}{t_4}$$

$$\frac{t_2}{t_1} = r \Rightarrow t_2 = t_1 r = ar$$

$$t_2 = a \times (r)^{(2-1)}$$

$$\frac{t_3}{t_2} = r \Rightarrow t_3 + t_2 r = ar \times r$$

$t_3 = a(r)^2 = a(r)^{(3-1)}$

$\frac{t_4}{t_3} = r \Rightarrow t_4 = t_3 r = a(r)^2 \times r$

$t_4 = a(r)^3 = a(r)^{(4-1)}$

$t_n = (r)^{(n-1)}$

The Geometric series upto n terms is generally written as

$a + ar\ a(r)^2 + a(r)^3\ .....+ar(r)^{n-1}$

4.2.2.PROPERTIES OF GEOMETRIC SERIES :

(a) If all the terms of a Geometric series be multipliced or divided by the same quantity, the resulting series is also a geometric series.

(b) The Resulting series, formed by taking the product of the corresponding terms of two geometic series is also a geometric series.

(c) The resulting series formed of the reciprocals of the corresponding terms of a geometcii series.

4.2.3. To find the sum to n terms of a GP :

Pooof : let a be the first terms and r be the common ratio, n be the number of the terms, Sn be the sum to n terms of a GP.

$S_n = a + ar + a(r)^2\ a(r)^3 + ... + a(r)^{n-2} + a(r)^{n-1}$ .....(1)

Multiplying both sides by (r) in (i)

$rS_n = ar + a(r)^2 + a(r)^3 + ... + a(r)^{n-2} \times r + a(r)^{n-1} \times r$

$rS_n = ar + a(r)^2 + a(r)^3 + ... + a(r)^{n-1} + a(r)^n$

Eqn (i) – Eqn (2) ....(2)

$S_n = a + ar + a(r)^2 + a(r)^3 + ... + a(r)^{n-1}$

$rS_n = ar + a(r)^2 + \quad ....a(r)^{n-1} + a(r)^n$

$- \quad - \quad\quad - \; - \quad - \quad - \quad -$

....

$Sn - rSn = a - a(r)^n$

$Sn(1-r) = a\left(1-(r)^n\right)$

$$S_n = \frac{a\left(1-(r)^n\right)}{1-r} 1 > r$$

Eqn (2) – Eqn (1)

$$rSn = ar + ar(r)^2 + a(r)^3 + ... + a(r)^{n-1} + a(r)^n$$

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....

$$rSn - Sn = -a + a(r)^n$$

$$Sn(r-1) = a(r)^n - a$$

$$Sn(r-1) = a\left((r)^n - 1\right)$$

$$S_n = \frac{a\left((r)^n - 1\right)}{r-1} |r > 1|$$

4.2.4. To find the sum to infinity terms of a geometric series, when the positive value of the common ratio is less than one i.e. proper fraction.

Proof : If the sum of n terms of a GP is denoted by S then

$$S = a + ar + a(r)^2 + ... + a(r)^{n-2} + a(r)^{n-1}$$

$$rSn = ar + a(r)^2 + ... + a(r)^{n-2} + a(r)^{n-1} + a(r)^n$$

– – – – – –

....

$S - rSn = a - a(r)^n$

$S(1 - r) = a(1 - (r)^n)$

$$S = \frac{a\left(1-(r)^n\right)}{1-r} = \frac{a}{1-r} = \frac{a(r)^n}{1-r}$$

Thus the sum of n terms of the G.P.

$$= \frac{a}{1-r} - \frac{a(r)^n}{1-r}$$

Here $|r| < |$ i.e. $-1 < r < |$

Hence as n gets larger and longer, $(r)^n$ becomes smaller and smaller and $(r)^n$ can be made as small in size, as we please by taking n sufficiently large. When n increases indefinitely $(r)^n$ gets closer to zero, since $-1 < r < 1$

Hence if $n \to \infty, (r)^n \to 0$

$$\therefore \frac{a(r)^n}{1-r} \to 0$$

Hence from (i)

The sum to infinite terms of a GP

$$\boxed{S_\infty = \frac{a}{1-r}}$$

4.2.5 GEOMETRIC MEAN (G.M.)

(a) Single Geometric Mean : If three terms are in geometric progression then the middle one is called the geometric mean between the other two.

Let a and b be the two given quantities and x be the single GM between them

$$\because \frac{x}{a} = \frac{b}{x} \Rightarrow x^2 = ab$$

$\boxed{x = \sqrt{ab}}$ Single GM

(b) To insert n gcometric means between two given quantities.

Proof : Let a and b be the two tiven quantities and $x_1, x_2, x, x_3......x_n$ be the n GMs between them.

$\therefore a, x_1, x_2, x_3.......x_n$, b are in GP

first term = a

last term = b

Total number of terms = n + 2

let r = common ratio

$$1 = t_n = a(r)^{n-1}$$

$$b = a(r)^{n+2-1}$$

$$\frac{b}{a} = (r)^{n+1}$$

$$\boxed{\left(\frac{b}{a}\right)^{\frac{1}{n+1}} = r}$$

$$x_1 = 2\text{nd term} = ar = a\left(a/b\right)^{\frac{1}{n+1}}$$

$$x_2 = 3\text{rd term} = a(r)^2 = a\left(b/a\right)^{\frac{2}{n+1}}$$

$$x_3 = 4\text{th term} = a(r)6 = a\left(b/a\right)^{\frac{3}{n+1}}$$

$$x_n = (n+1)\text{th term} = a(r)^3 n = a\left(b/a\right)^{\frac{n}{n+1}}$$

4.2.6 ILLUSTRATION:

Write down the fifth term of the series

4 + 12 + 36 + ...

Soln. $a = 4, r = \frac{12}{4} = 3, \; \boxed{n = 5}$

$$t_n = a(r)^{n-1}$$

$$t_5 = 4(3)^{5-1} = 4(3)^4 = 4(9 \times 9)$$

$$t_5 = 4 \times 81 = 324$$

$$t_5 = 324$$

4.2.7. Find the sum of the following series upto n terms and also find sum upto 5 terms.

$$\frac{1}{4} - \frac{1}{2} + 1 - 2 + 4$$

Soln. $a\frac{1}{4}, r = \dfrac{\frac{-1}{2}}{\frac{1}{4}} = \frac{-1}{2} \times \frac{4}{1} = -2$

1 > r

$$S_n = \frac{a\left(1-(r)^n\right)}{1-r}$$

$$S_n = \frac{\frac{1}{4}\left\{1-(-2)^n\right\}}{1-(-2)} = \frac{\frac{1}{4}\left\{1-(-2)^n\right\}}{3}$$

$$S_n = \frac{1}{4\times 3}\left\{1-(-2)^n\right\} = \frac{1}{12}\left\{1-(-2)^n\right\}$$

$$S_5 = \frac{1}{12\left\{1-(-2)^5\right\}} = \frac{1}{12}\left\{1-(-32)\right\}$$

$$S_5 = \frac{1}{12}(1+32) = \frac{33}{12} = \frac{11}{4}$$

4.2.8. The third of a GP is 12 and 6th term is 96. Find the sum of Nine terms.

Soln. We know that

$$t_n = a(r)^{n-1}$$

$$t_3 = a(r)^{3-1} = a(r)^2 = 12$$

$$t_6 = a(r)^{6-1} = a(r)^5 = 96$$

$$\frac{\text{Eqn}(i)}{\text{Eqn}(ii)} \Rightarrow \frac{a(r)^2}{a(r)^5} = \frac{12}{96}$$

$$\frac{1}{(r)^{5-2}} = \frac{1}{8}$$

$$(r)^3 = 8 = (2)^3$$

$$r = 2$$

From (i)

$$a(r)^2 = 12$$

$$a(2)^2 = 12$$

$$\boxed{a = \frac{12}{4} = 3}$$

$$\boxed{r > 1}$$

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$

$$S_9 = \frac{3\left((2)^9 - 1\right)}{2-1} = \frac{3(512-1)}{1}$$

$$S_9 = 3 \times 511 = 1533$$

4.2.9. Find the sum upto n terms

S = 1.4 + 3.04 + 5.004 + 7.0004 + upto n terms

Soln. S = 1.4 + 3.04 + 5.004 + 7.0004 upto n terms

S = (1 + 0.4) + (3 +0.04) + (5 +0.004) + upto n terms

$S = \{1 + 3 + 5 + \text{upto n terms}\}$ +(0.4 + 0.04 + 0.004 + 0.0004 + upto n terms)

$$S = \{1 + 3 + 5 + \text{upto n terms}\} + \left(\frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \text{upton terms}\right)$$

$$S = \frac{n}{2}\left(2 + (n-1)d\right) + \frac{a\left(1 - r^n\right)}{1-r}$$

$$S = \frac{n}{2}\left(2 \times 1(n-1)2\right) + \frac{4}{10}\frac{\left(1 - \left(\frac{1}{10}\right)^n\right)}{1 - \frac{1}{10}}$$

$$S = \frac{n}{2}\left(2 + 2n - 2\right) + \frac{4}{10}\left(\frac{1 - \left(\frac{1}{10}\right)^n}{9/10}\right)$$

$$S = n^2 + \frac{4}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)$$

4.2.10 Sum upto n terms

S = 3 + 33 + 333 + upto n terms

Soln. S = 3 + 33 + 333 + upto n terms

S = 3 {1 + 11 + 111 + upto n terms}

$$S=\frac{3}{9}\{9+99+999+\text{upto n terms}\}$$

$$S=\frac{3}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\text{upto n terms}\right]$$

$$S=\frac{3}{9}\left[\left(10+10^2+10^3+\text{upto terms}\right)-n\right]$$

$$S=\frac{3}{9}\left[\frac{a\left(r^n-1\right)}{r-1}-n\right]$$

$$S=\frac{3}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]=\frac{3}{9}\left[\frac{10}{9}\left(10^n-1\right)-n\right]$$

NOTE : (a) When three consecutive terms are in GP

$$\frac{a}{r}, a, ar$$

(b) When four consecutive terms are in GP.

$$\frac{a}{r^3}, \frac{a}{r}, ar, a(r)^3$$

4.2.11.Th product of three consecutive terms in a GP is 216 and their sum is 19thn find the terms.

Soln. Let $\frac{a}{r}, a, ar$ be the three consecutive terms of GP.

A/c of Question.

$$\frac{a}{r}\times a\times ar=216$$

$$(a)^3=216=(6)^3$$

$$(a)^3=(6)^3$$

a = 6

And their sum is 19

$$\frac{a}{r}+a+ar=19$$

$\Rightarrow \frac{6}{r} + 6 + 6r = 19$

$\Rightarrow \frac{6}{r} + 6r = 19 - 6$

$\Rightarrow \frac{6 + 6r^2}{r} = 13$

$\Rightarrow 6r^2 + 6 = 13r$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow 6r^2 - 9r - 4r + 6 = 0$

$\Rightarrow 3r(2r - 3) - 2(2r - 3) = 0$

$\Rightarrow (3r - 2)(2r - 3) = 0$

3r − 2 =	2r − 3 = 0
3r = 2	2r = 3
r = 2/3	r = 3/2
a = 6	$\frac{a}{r} = \frac{\frac{6}{2} \times 3}{2} = 9$
r = 2/3	a = 6
	$ar = 6 \times \frac{2}{3} = 4$
	9, 6, 4

4.2.12. If the continued product of three number in G.P. is 216 and the sum of their product in pairs is 156, find the numbers.

Soln. Let the three numbers in GP be

$\frac{a}{r}, a, ar$

$\frac{a}{r} \times a \times ar = 216$

$(a)^3 = 216 = (6)^3$

a = 6

By the Questions

$$\frac{a}{r}\times a+\frac{a}{r}\times ar+a.ar=156$$

$$\Rightarrow \frac{a^2}{r}+a^2+a^2r=156$$

$$\Rightarrow a^2\left(\frac{1}{r}+1+r\right)=156$$

$$\Rightarrow (6)^2\left\{\frac{1+r+r^2}{r}\right\}=156$$

$$\Rightarrow 36\left\{\frac{1+r+r^2}{r}\right\}=156$$

$$3\left(r^2+r+1\right)=13r$$

$$\Rightarrow 3r^2+3r+3=13r$$

$$\Rightarrow 3r+3r-13r+3=0$$

$$\Rightarrow 3r^2-10r+3=0$$

$$\Rightarrow 3r-9r-r+3=0$$

$$\Rightarrow 3r(r-3)-1(r-3)=0$$

$$\Rightarrow (3r-1)(r-3)=0$$

$r - 3 = 0$ $\quad$ $3r - 1 = 0$

$r = 3$ $\quad$ $3r = 1$

$$r=\frac{1}{3}$$

Hence the three numbers be

When $r = 3$ $\quad$ when $r=\frac{1}{3}$

$\frac{a}{r}=\frac{6}{3}=2$ $\quad$ $\frac{a}{r}=\frac{6}{1}=18$

$a = 6$ $\quad$ $a = 6$

$ar = 6 \times 3 = 18$ $\quad$ $ar = 6 \times \frac{1}{3}=2$

18, 6, 2