M C A

PART - I

SUBJECT
DISCRETE MATHEMATICS

PAPER - III

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SET, RELATIONS AND FUNCTIONS

Theorem of total Probability or Rule of Addition

Summary

Questions for Exercise

Theorem of Total Probability

or,

Rule of Addition

Theorem : If A and B be two mutually exclusive events then $P(A \cup B)=P(A)+P(B)$ i.e. the probability that either A or B occurs is the sum of the probabilities of the event A and B.

Proof. : Let the total number of all possible elementary events be N out of which m_1 are favourable to A and m_2 are favourable to B. Then the number of elementary events that are favourable to either A or B is $m_1 + m_2$ (since A and B are two mutually exclusive events)

Hence $P(A \cup B)=\text{Probability that either A or B occurs}$

$$\frac{m_1 \quad m_2}{N} \quad \frac{m_1}{N} \quad \frac{m_2}{N} = P(A) + P(B)$$

$$P(A \cup B)=P(A)+P(B)$$

Extension :

$$P(A_1 \cup A_2 \cup A_3 ...... \cup A_r)$$

$$=P(A_1)+P(A_2) \quad P(A_3)+ .... P(A_r)$$

Corollary : If A and B are two events, not mutually exclusive, connected with a random experiment then A will occur if and only if any one of the mutually execlusive events $A \cap B$ and $A \cap B'$ occurs.

$$P(A)=P(A \cap B)+P(A \cap B')$$

$$P(B)=P(A \cap B)+P(A' \cap B)$$

Theorems : If A and B be any two events (not necessarly mutually exclusive) then $P(A \cup B)$ ie the probability that at least one of the two events A and B occurs is given by

$$P(A \cup B)=P(A)+P(B)-(A \cap B)$$

Proof. : Let S be the sample space and A, B be any two events of S

$$P(A \cup B)=\frac{n(A \cup B)}{n(s)}$$

where $n(A \cup B)$ = the nubmber of sample points in $A \cup B$ and n(S) = The total no. of all possible sample points in S.

$$\frac{\text{From Set theory we know that}}{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

$$P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

$$P(A \cup B) = \frac{n(A)}{n(S)} \quad \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\boxed{P(AUB) = P(A) + P(B) - P(A \cap B)}$$

Generalisation : We know that for any three events A, B, C

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$$

$$-n(A \cap C) - n(B \cap C) \quad n(A \cap B \cap C)$$

$$P(A \cup B \cup C) = \frac{n(A \cup B \cup C)}{n(S)}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$$

$$-P(B \cap C) + P(A \cap B \cap C)$$

Question : From a set of 17 balls marked 1, 2, 3, 4...... 16, 17. One ball is drawn at random

(a) What is the chance that its number is a multiple of 3 or 7 (b) What is the probability that its number is an even number greated than 9 ?

Soln. : Let S be he sample space connected with the drawing

Then $S \quad \{1, 2, 3, ...17\}, n(S) \quad 17$

(a) Let A be the event that the number of the ball is a multiple of 3 and B be the events that the number of ball is multiple of 7.

Then A = {3, 6, 9, 12, 15} n(A) = 5, B = {7, 14} n(B) = 2

$$P(A)=\frac{n(A)}{n(S)} \quad \frac{5}{17}$$

$$n(B)=\frac{n(B)}{n(S)} \quad \frac{2}{17}$$

Since A and B are mutually exclusive events by the theorem of total probability

$$P(A\cup B)=P(A)+P(B)$$

$$\frac{5}{17} \quad \frac{2}{17} \quad \frac{7}{17}$$

(b) Let C be the event that the numbers of ball is an even number greater than 9.

Then C = { 10, 12, 14, 16}

$$P(C)=\frac{n(C)}{n(S)} \quad \frac{4}{17}$$

Ques. : The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting atleast one contract is 4/5, what is the probability that he will get both the contract ?

Soln. : Let A be the event that the contractor will get the plumbing contract and B, That he will get that electric contract.

$$P(A)=2/3, P(B')=5/9, P(A\cup B)=4/5$$

$$P(B)=1-P(B')=1-5/9=4/9$$

By the theorem of total probability

we have

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\frac{4}{5} \quad \frac{2}{3} \quad \frac{4}{9}-P(A\cap B)$$

$$P(A\cap B)=\frac{2}{3}\quad\frac{4}{9}-\frac{4}{5}\quad\frac{14}{45}$$ Ans.

A and B are two events not mutually exclusive, connected with a random experiment E. If $P(A)=\frac{1}{4}, P(B)=2/5$ and $P(A\cup B)=\frac{1}{2}$

Find the value of the following Probability

(i) $P(A\cap B)$ (ii) $P(A\cap B')$ (iii) $P(A'\cup B')$

Soln. We know that

(i) $P(A\cup B)\ =P(A)+P(B)–P(A\cap B)$

$$\frac{1}{2}\quad\frac{1}{4}\quad\frac{2}{5}-P(A\cap B)$$

$$P(A\cap B)=\frac{1}{4}\quad\frac{2}{5}-\frac{1}{2}\quad\frac{3}{20}$$

(ii) Even A will occur if and only if any one of the two mutually exclusive events $A\cap B$ and $A\cap B'$ occurs $\qquad P(A)=P(A\cap B)+P(A\cap B')$

$$\frac{1}{4}\quad\frac{3}{20}\quad P(A\cap B')$$

$$\frac{5\quad 3}{20}\quad\frac{1}{10}\quad P(A\cap B')$$

(iii) By Demorgan's Law

$$(A\cap B)'=A'\cup B'$$

$$P(A'\cup B')=P[(A\cap B)']=1–P(A\cap B)$$

$$P(A'\cup B')=1-\frac{3}{20}\quad\frac{17}{20}$$ Ans.

Summary

(1) In Combinatorial analysis we intend to determine the number of logical possibilities of occurrence of events without looking into individual cases. If two tasks T_1 and T_2 can be performed in n_1 and n_2 ways, respectively and these are not to be performed simultaneously then two task T_1 and T_2 can be performed in $n_1 + n_2$ ways. This is the rule of Addition. In set theoretical notation, the rule of Addition can be interpreted as follows.

$$n(A \cup B) = n(A) + n(B)$$

In set theoretical notation, the rule of multiplication can be interpreted as follows.

$$n(A \times B) = n(A) \times n(B)$$

(2) The different selections, that can be obtained by taking the same number r of things from a given collection of n different things without any regard to the order of the things, are called the combination of n things taken r at a time and the number of such combination is denoted by ${}^{n}c_{r}$. The different arrangements that can be obtained by taking the some number r of things from a given collection of *n* different things, are called permutations of n things taken r at a time and the number of such permutations is denoted by ${}^{n}p_{r}$.

(3) There are number of objects which are not all different, some are of one kind, others are of second kind and yet some others are of a third kind and like that. Example of such a kind is often found when we come across words when certain letters are repeated.

In such a case if among the n things, p things of them are of one kind, q things of them are of second kind, and so on such that $p + q + r + \ldots = n$,

The number of permutations of n things taken together when all the things are not different is given by $\dfrac{\underline{|n}}{\underline{|p}\,\underline{|q}\,\underline{|r}}$

Since number of permutations are reduced by $\underline{|p}\,\underline{|q}\,\underline{|r}\ldots\ldots$ from the original permutation when things were all different.

(4) The coefficients ${}^nc_0$, ${}^nc_1$, ${}^nc_2$, ... in. The expansion of a Binomial are called Binomial Coefficients.

(5) Permutations and combinations have an important role in the computations of probabilities. The number of distinct linear arrangement with n different objects, taking all together is $\underline{|n}$.

Questions for Exercise

(1) Find the value of r if

9P5 + 5 × 9P4 = 10Pr

(2) If ${}^{2n}P_3 \quad 100 \quad {}^{n}P_2$ find the value of n

(3) Prove that

(a) $\underline{|2n}\,\underline{|n-1} \quad 2\underline{|n}\,\underline{|2n-1}$

(b) ${}^nP_r \quad (n-r \quad 1)\,{}^nP_{r-1}$

(c) ${}^nP_r \quad {}^{n-1}P_r \quad r \quad {}^{n-1}P_{r\ 1}$

(4) In how many different ways can be letters of the word Examination be arranged ?

(5) In How many ways can 4 books of Econmics and 2 books of Mathematics be arranged on a shelf so that all books of the same subject are put together ?

(6) How many numbers can be formed from the digits 1, 2, 3, 5, 7, 9 so that they are not divisible by 5.

(7) In how many ways 7 boys and 5 girld can be arranged in a row so that no two girls are together ?

(8) If ${}^nP_r \quad 6720$ and ${}^nc_r \quad 56$ then find the value of n and r.

(9) If ${}^{18}c_r \quad {}^{18}c_{r-2}$ find ${}^rc_6$.

(10) In How many ways 9 toys can be divided among 4 children if the youngest one gets Three and the remaining children get two boys each.

(11) In how many ways a committee of 5. Persons can be formed out of 7 Indians and 4 Pakistan is when the committee contains

(a) All Indians (b) At least two Pakistani (c) At least two Indians and two Pakistani. (d) not more than two Pakistan.

(12) Expand

(a) $5-\frac{x}{6} \quad ^{6}$ (b) $\frac{3x}{2}-\frac{y}{6} \quad ^{6}$ (c) $(1-2x \quad 3x^{2})^{3}$

(d) $(\sqrt{x \quad 1}-\sqrt{x-1})^{6} \quad (\sqrt{x \quad 1} \quad \sqrt{x-1})^{6}$

(13) Find the Independent term of x.

(a) $x^{2} \quad \frac{2}{x^{3}} \quad ^{15}$ (b) $(1 \quad x)^{p} \quad 1 \quad \frac{1}{x} \quad ^{q}$ (c) $x^{2} \quad \frac{2}{x^{2}} \quad ^{8}$

(d) $\sqrt{x} \quad \frac{3}{x^{2}} \quad ^{10}$

(14) Find the middle term in the Expansion of Binomial Expression.

(a) $x \quad \frac{1}{2x} \quad ^{10}$ (b) $\frac{3}{x^{2}}-\frac{x^{3}}{6} \quad ^{6}$ (c) $3x-\frac{x^{3}}{6} \quad ^{9}$

(d) $2x \quad \frac{x^{2}}{4} \quad ^{13}$

(15) Find the coeffieicnt of x^{-6} in the Expansion of $\frac{3x^{2}}{2}-\frac{1}{3x} \quad ^{9}$

(16) Find the value of n if the coefficient of x^7 and x^8 are equal in the expansion of $3 \quad \frac{x}{2} \quad ^{n}$.

(17) If the three consecutive coefficients in the expansion of $(1 \quad x)^{n}$ are 462, 330 and 165, Find n.

(18) Find the fifth term from the end in the expansion of $x \quad \frac{1}{x} \quad ^{12}$.

(19) Find the value of the following using Binomial theorem.

(a) $(999)^3$ (b) $(1.01)^5$ (c) $(101)^4$ (d) $(11)^9$.

(20) Find the probability that in a game of bridge a hand of 13 cards will contain all the 4 aces.

(21) What is the probability of getting 3 white balls in draw of 3 balls from a box containing 5 white and 4 black balls .

(22) A sample of 3 items is selected at random from a box of 12 items of which 3 are defective. Find the possible number of defective combinations of the said 3 selected items along with probability of defective combinations.