M C A

PART - I

SUBJECT
DISCRETE MATHEMATICS

PAPER - III

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SET, RELATIONS AND FUNCTIONS

Binomial Theorem

Probability

Binomial Theorem

Binomial Expression :

An algebraic expression consisting of only two terms is called a Binomiasl expression.

Example :

(i) $x + y$ (ii) $4x - 3y$ (iii) $x^2 + y^2$ (iv) $x^2 \quad \frac{1}{a^2}$

Binomial Theorem :

The formula by which any power of Binomial expression can be expanded in the form of a series is known as Binomial theorem. This theorem was given by sir issac Newton.

Binomial Theorem for a positive integral index.

Statement : If n is a positive integer $(a \quad b)^n$

$$^n c_0 (a)^{n-0} b^0 \quad ^n c_1 (a)^{n-1} b^1 \quad ^n c_2 (a)^{n-2} b^2$$

$$^n c_3 (a)^{n \; 3} b^3 \quad .... \quad ^n c_n (a)^{n-n} b^n$$

Pascal's Triangle

Binomial coefficunts can be found with the help of Pascal's Triangle by the use of

$$^{n \; 1}c_r \quad ^n c_{r-1} \quad ^n c_r$$

For $^n c_r, 0 \quad r \quad n \quad 10,$ The Pascal's triangle is

n	
0	1
1	1 1
2	1 2 1
3	1 3 3 1

4 1 4 6 4 1

5 1 5 10 10 5 1

6 1 6 15 20 15 6 1

7 1 7 21 35 35 21 7 1

8 1 8 28 56 70 56 28 8 1

9 1 9 36 84 126 126 84 36 9 1

10 1 10 45 120 210 252 210 120 45 10 1

Expand

$(x + 2)^4$

Solutions :

$$(x \quad 2)^4 \quad {}^4c_0(x)^{4-0}(2)^0 \quad {}^4c_1(x)^{4\ 1}(2)^1$$

$$^4c_2(x)^{4\ 2}(2)^2 \quad {}^4c_3(x)^{4\ 3}(2)^3 \quad {}^4c_4(x)^{4-4}(2)^4$$

$$x^4 \quad \frac{\underline{\lfloor 4}}{\underline{\lfloor 1}\ \underline{\lfloor 4-1}}(x)^3(2) \quad \frac{\underline{\lfloor 4}}{\underline{\lfloor 2}\ \underline{\lfloor 4-2}}(x)^2 \quad 4$$

$$\frac{\underline{\lfloor 4}}{\underline{\lfloor 3}\ \underline{\lfloor 4-3}}(x)(8) \quad \frac{\underline{\lfloor 4}}{\underline{\lfloor 4}\ \underline{\lfloor 4-4}}(x)^0 \quad 16$$

$$x^4 \quad 8x^3 \quad 24x^2 \quad 32x \quad 16$$

Expand

$(x^2 \quad 4)^3$

Solutions :

$$(x^2 \quad 4)^3 \quad \{x^2 \quad (-4)^3$$

$$(x^2 \quad (\ 4)\}^3$$

$$= {}^3c_0(x^2)^{3\ \ 0}(\quad 4)^0 \quad {}^3c_1(x^2)^{3\ \ 1}(\quad 4)^1$$

$${}^3c_2(x^2)^{3\ \ 2}(-4)^2 \quad {}^3c_3(x^2)^{3\ \ 3}(-4)^3$$

$$(x^2)^3 \quad \frac{\lfloor 3}{\lfloor 1 \lfloor 3-1}(x^2)^2(-4) \quad \frac{\lfloor 3}{\lfloor 2 \lfloor 3-2}(x^2) \quad 8 \quad (-64)$$

$$x^6 \quad \frac{3 \quad \lfloor 2}{\lfloor 1 \lfloor 2}(x^4)(-4) \quad \frac{3 \quad \lfloor 2}{\lfloor 2 \quad \lfloor 1} \quad x^2 \quad 8-64$$

$$x^6 - 12x^4 \quad 24x^2 - 64$$

Questions Find the 4th term in the expansion of $(2x + 3y)^5$

Solutions :

$$\boxed{\begin{array}{l} Rules \\ (a \quad b)^n \\ (r \quad 1)th\,term \quad {}^n c_r(a)^{n\ \ r} b^r \end{array}}$$

4th term = (3 + 1) th term

$${}^5c_3(2x)^{5-3}(3y)^3$$

$${}^5c_3(2x)^2(3y)^3$$

$$\frac{\lfloor 5}{\lfloor 3 \lfloor 5-3}(2x)^2(27y^3)$$

$$\frac{5 \quad 4 \quad \lfloor 3}{\lfloor 3 \quad 2 \quad 1} \quad 108x^2y^3 \quad 1080x^2y^3 \text{ Ans.}$$

Rules

To find the independent term of x.

To find the term independent of x we. let (r + 1)th term be the free from x. In this term by equating the powers of x to zero, we find the value of r.

Question : Find the term independent of x in the expansion $\frac{3x^2}{2} \quad \frac{1}{3x} \quad ^{9}$

Solutions :

$$\frac{3x^2}{2} \quad -\frac{1}{3x} \quad ^{9}$$

Let (r + 1) th term be the Independent of x.

(r + 1)th term $\;{}^{9}c_r \; \frac{3x^2}{2}^{\,9\ r} \quad -\frac{1}{3x}^{\,r}$

$${}^{9}c_r \; \frac{3}{2}^{\,9\ r} (x^2)^{9\ r} \quad -\frac{1}{3}^{\,r} \quad \frac{1}{x}^{\,r}$$

$${}^{9}c_r \; \frac{3}{2}^{\,9\ r} (x)^{18\ \ 2r} \quad -\frac{1}{3}^{\,r} \quad x^{\ 1\ \ r}$$

$${}^{9}c_r \; \frac{3}{2}^{\,9\ r} (x)^{18\ \ 2r} \quad -\frac{1}{3}^{\,r} \quad x^{\ r}$$

$${}^{9}c_r \; \frac{3}{2}^{\,9\ r} (x)^{18\ \ 2r\ \ r} \quad -\frac{1}{3}^{\,r}$$

Equating the power of x equal to zero

18 – 3r = 0

3r = 18

r = 6

$${}^{9}c_{6}\ \frac{3}{2}^{9\ \ 6}\ \ \frac{1}{3}^{6}$$

$${}^{9}c_{6}\ \frac{3}{2}^{3}\ \ \frac{1}{3}^{6}$$

$$\frac{\underline{|9}}{\underline{|6}\ \underline{|9-6}}\ \ \frac{3\quad 3\quad 3}{2\quad 2\quad 2}\qquad \frac{1}{3\quad 3\quad 3\quad 3\quad 3\quad 3}$$

$$\frac{9\quad 8\quad 7\quad \underline{|6}}{\underline{|6}\quad 3\quad 2\quad 1}\ \ \frac{1}{8}\ \ \frac{1}{27}\ \ \frac{7}{18}\ \text{Ans.}$$

Question. How many terms are there in the expansion of $(2x \times 3y)^5$. Find the middle term.

Solutions : Here n = 5, so number of terms = 5 + 1 = 6 and Here are sec that n is odd then there are two middle terms.

$$\frac{n\quad 1}{2}\ \text{ and }\ \frac{n\quad 3}{2}\ \text{ th term}$$

$$\frac{5\quad 1}{2}\ \text{ th and }\ \frac{5\quad 3}{2}\ \text{ th term = 3rd term and 4th term}$$

3rd term = (2 + 1)th term

$${}^{5}c_{2}(2x)^{5-2}(3y)^{2}$$

$${}^{5}c_{2}(2x)^{3}(3y)^{2}$$

$$\frac{\underline{|5}}{\underline{|2}\ \underline{|5-2}}(2x)^{3}(9y^{2})$$

$$\frac{5\quad 4\quad \underline{|3}}{2\quad 1\quad \underline{|3}}\ \ 8x^{3}\quad 9y^{2}$$

$$720x^3y^2$$

4the term = (3 + 1)th term

$$^5c_3(2x)^{5-3}(3y)^3$$

$$\frac{\lfloor 5}{\lfloor 3 \lfloor 5-3}(2x)^2(27y^3)$$

$$\frac{5 \quad 4 \quad \lfloor 3}{\lfloor 3 \quad 2 \quad 1} \quad 4x^2 27y^3$$

$$1080x^2y^3$$

Rule for finding middle terms.

$$(a \quad b)^n$$

Case I When $\boxed{n \text{ is even}}$ then there is only one middle term

$$(\frac{n}{2} \quad 1)th \text{ term}.$$

Case II When $\boxed{n \text{ is odd}}$ then there are two middle terms $\frac{n \quad 1}{2}$ and $\frac{r \quad 3}{2}$ th term

Question Prove that the middle term in the expansion of $(1 + x)^{2n}$ is $\frac{1.3.5...(2n \quad 1)}{\lfloor n}(2)^n.x^n$ n being positive integer.

Proof : $(1 \quad x)^{2n}$

Here we see that n is even because when we put n = 1, 2, 3......

2 ×1 = 2

$2 \times 2 = 4$

$2 \times 3 = 6$

$\frac{n}{2} \quad 1 \;\; th$ term $\quad \frac{2n}{2} \quad 1 \;\; th$ term = (n + 1)th term

$(n \quad 1)th$ term $\quad {}^{2n}c_n (1)^{2n-n} (x)^n$

$${}^{2n}c_n (1)^{2n-n} (x)^n = {}^{2n}c_n (x)^n$$

$$\frac{\lfloor 2n}{\lfloor n \lfloor n} (x)^n$$

$$\frac{2n(2n-1)(2n-2)(2n-3).. \quad 4 \quad 3 \quad 2 \quad 1 \quad x^n}{\lfloor n \quad \lfloor n}$$

$$\frac{\{2n(2n-2)(2n-4)....4 \quad 2| \quad \{(2n-1)(2n-3)....3 \quad 1| \, x^n}{\lfloor n \quad \lfloor n}$$

$$\frac{\{2 \quad n \quad 2(n-1) \quad 2(n-2)...2 \quad 2 \quad 2 \quad 1\}\{(2n-1)(2n-3)...3 \quad 1)x^n}{\lfloor n \quad \lfloor n}$$

$$\frac{(2)^n\{n(n-1)(n-2)......2 \quad 1\}\{(2n-1)(2n-3)....3 \quad 1)x^n}{\lfloor n \quad \lfloor n}$$

$$\frac{(2)^n\{(2n-1)(2n-3)....3 \quad 1\}x^n}{\lfloor n} \text{ Ans.}$$

If the absolute term in the expansion of $\sqrt{x} - \frac{k}{x^2} \quad ^{10}$ is 405, find the value of k.

Solutions : By the absolute term, we mean the term Independent of x (also called the constant term). So suppose Tr+1 is the absolute term in the expansion of

$$\sqrt{x}-\frac{k}{x^2}\;^{10} \qquad \sqrt{x} \qquad \frac{k}{x^2}\;^{10}$$

Tr+1th term $\;{}^{10}c_r(\sqrt{x})^{10\ \ r}\ \ \frac{-k}{x^2}^{\,r}$

$$ {}^{10}c_r\,(x)^{\frac{1}{2}(10\ \ r)}\quad \frac{(-k)^r}{x^{2r}}$$

$$ {}^{10}c_r.(x)^{\frac{10\ \ r}{2}}\,(-k)^r\,(x\ \ ^2)^r$$

$$ {}^{10}c_r\,(x)^{\frac{10\ \ r}{2}\ \ 2r}\,(-k)^r$$

$$ {}^{10}c_r\,(x)\frac{10-r-4r}{2}(-k)^r$$

$$\frac{10-5r}{2}\quad 0$$

5r = 10, r = 2

T(1+2)th term $=(-k)^2\quad {}^{10}c_2$

$$405\quad k^2\quad \frac{|\underline{10}}{|\underline{10\quad 2}}\quad \frac{1}{|\underline{2}}$$

$$405\quad \frac{k^2\quad 10\quad 9\quad |\underline{8}}{|\underline{8}}\quad \frac{1}{|\underline{2}}$$

$2\quad \frac{405}{9\quad 10}\quad k^2$ | these are the required value of k.

$k^2 \quad 9$

$k \quad 3$

Find the 9th term in the expansion of $\sqrt{a} - 2\sqrt{b}$ ^{12}

Solution : 9th term = (8 + 1)th term $\quad \sqrt{a} \quad (\quad 2\sqrt{b})$ ^{12}

$$^{12}c_8(\sqrt{a})^{12-8}(-2\sqrt{b})^8$$

$$^{12}c_8(\sqrt{a})^4(-2\sqrt{b})^8$$

$$\frac{\underline{|12}}{\underline{|8}\,\underline{|12-8}}(a^2)(2)^8 b^4$$

$$\frac{12 \quad 11 \quad 10 \quad 9 \quad \underline{|8}}{\underline{|8} \quad 4 \quad 3 \quad 2 \quad 1} \quad a^2b^4 \quad (2)^8 \quad 55 \quad 9 \quad 256.a^2b^4$$

$495 \quad 256.a^2b^4$ Ans.

To find coefficient of x^m.

To find the coefficient of any power of *x* in $(x + a)^n$. We assume the required term as (r + 1)th term and in this term by equating, power of x to the given power, we gets the value of r. Then by putting the value of r in (r + 1) th term, we get the required coefficient.

Question : Find the coefficient of x^{32} and x^{-17} in the expansion $\quad x^4 - \frac{1}{x^3}$ ^{15}

Solution : Let (r + 1)th term be coefficient of x^{32} and x^{-17}

Now the Expression can be written in the Binomial form

$$x^4 - \frac{1}{x^3} \quad ^{15} \qquad x^4 \qquad -\frac{1}{x^3} \quad ^{15}$$

(r + 1)th term $\quad {}^{15}c_r(x^4)^{15-r} \quad -\frac{1}{x^3} \quad {}^{r}$

$${}^{15}c_r(x)^{60\ \ 4r}\frac{(\ \ 1)^r}{x^{3r}}$$

$${}^{15}c_r(x)^{60\ \ 4r}(-1)^r \quad x^{\ \ 3r}$$

$${}^{15}c_r(x)^{60\ \ 4r-3r}(-1)^r$$ ----------------- (1)

Equating the powers of x to equat to 32

$$60-7r \quad 32$$
$$60 \quad 32 \quad 7r$$
$$28 \quad 7r$$
$$r \quad 4$$

puting $r \quad 4$in (i)

$${}^{15}c_4(-1)^4$$
$$\frac{\lfloor 15}{\lfloor 4\ \lfloor 15-4}(\ \ 1)^4 \quad \text{Ans}$$

60 – 7r = – 17

60 + 17 = 7r

77 = 7r, r = 11

Putting r = 11 in (i)

$${}^{15}c_{11}(-1)^{11}$$

$$\frac{\lfloor 15}{\lfloor 11\ \lfloor 15-11}(-1)^{11} \quad \frac{15 \quad 14 \quad 13 \quad 12 \quad \lfloor 11}{\lfloor 11 \quad \lfloor 4}(-1)^{11}$$

$$-\frac{15 \quad 14 \quad 13 \quad 12}{4 \quad 3 \quad 2 \quad 1}$$

$= -105 \times 13 = -1365$ Ans.

In the expansion of $(1 + x)^{43}$, the coefficient of (2r + 1)th term is equal to the coefficient of (r + 2)th term, find the value of r.

Solution : In $(1 + x)^{43}$

Coefficient of (2r + 1)th term =

$= {}^{43}c_{2r}\,(1)^{43-2r}\,(x)^{2r}$ ---------(i)

Coefficient of (r + 2)th term

$= \{(r+1) + 1\}$th term

$= {}^{43}c_{r+1}(1)^{43-(r+1)}\,(x)^{r+1}$

$= {}^{43}c_{r+1}\,(x)^{r+1}$ -------- (ii)

A/c to Question

Coefficient of (2r + 1)th term

=(r + 2)th term

${}^{43}c_{2r} = {}^{43}c_{r+1}$

we know that

$$\boxed{{}^{n}c_{r} \quad {}^{n}c_{n\ r}}$$

$${}^{43}c_{2r} \quad {}^{43}c_{43} \quad (r \quad 1)$$

$2r = 43 - (r + 1)$

$2r + r + 1 = 43$

$3r = 42$

$r = 14$

Question : The first three term in the expansion of $(1 + ax)^n$ are $1 + 12x + 64x^2$. Find the value of n and a.

Solutions : The first three terms in the expansion of $(1 \quad ax)^n$

$${}^{n}c_{0}(1)^{n-0}(ax)^{0} \quad {}^{n}c_{1}(1)^{n\ 1}(ax)^{1} \quad {}^{n}c_{2}(1)^{n\ 2}(ax)^{2}$$

$$1 \quad 12x \quad 64x^{2}$$

$$1 \quad \frac{\underline{|n}}{\underline{|1}\,\underline{|n \quad 1}}ax \quad \frac{\underline{|n}}{\underline{|2}\,\underline{|n-2}}a^{2}x^{2}$$

$$1 \quad 12x \quad 64x^{2}$$

$$= 1 \quad nax \quad \frac{n(n \quad 1)}{2 \quad 1} \quad a^{2}x^{2}$$

$$1 \quad 12x \quad 64x^{2}$$

Coefficien at x	$n \quad \frac{12}{a}$..................(i)
$na \quad 12$	

Coefficient of x^2

$$\frac{n(n \quad 1)}{2}a^{2} \quad 64$$

$$\frac{12}{a} \quad \frac{12}{a}-1 \;\; a^{2} \quad 64 \quad 2$$

$$\frac{12 \quad a}{a} \;\; a \quad \frac{64 \quad 2}{12}$$

$$12-a \quad \frac{64}{6}$$

$$12 \quad a \quad \frac{32}{3}$$

$$12 - \frac{32}{3} \quad a$$

$$\boxed{a \quad \frac{36-32}{3} \quad 4/3}$$

from (i) $n \quad \frac{12}{a} \quad \frac{12}{4} \quad 3,$ n = 9

In the Expansion of (1 + x)^n, three consecutive coefficients are 56, 70 and 56. Find the value of n and the position of the coefficient.

Solution : let three consecutive terms be (r+1)th, (r+2) and (r+3)th term

$$(r+1)\text{th} = {}^{n}c_{r} \quad 56 \text{------ (i)}$$

$$(r+2)\text{th term} = {}^{n}c_{r\ 1} \quad 70 \text{ ------- (ii)}$$

$$(r+3)\text{th term} = {}^{n}c_{r\ 2} \quad 56 \text{ ---------(iii)}$$

$$\frac{Equn(i)}{Equn\ (ii)} \quad \frac{{}^{n}c_{r}}{{}^{n}c_{r\ 1}} \quad \frac{56}{70}$$

$$\frac{\frac{\underline{|n}}{\underline{|r}\ \underline{|n-r}}}{\frac{\underline{|n}}{\underline{|r\quad 1}\ \underline{|n-(r\quad 1)}}} = \frac{4}{5}$$

$$\frac{\underline{|r\quad 1}\underline{|n\quad r-1}}{\underline{|r}\ \underline{|n-r}} \quad \frac{4}{5}$$

$$\frac{(r\quad 1)\underline{|r}\underline{|n\quad r\quad 1}}{\underline{|r}\ (n\quad r)\ \underline{|n\quad r\quad 1}} \quad \frac{4}{5}$$

$$\frac{r \quad 1}{n \quad r} \quad \frac{4}{5}$$

5(r + 1) = 4(n – r)

5r + 5 =4n – 4r

9r = 4n – 5 -------------- (iv)

$$\frac{Eqn\ (ii)}{Eqn\ (iii)} \qquad \frac{{}^{n}c_{r \quad 1}}{{}^{n}c_{r \quad 2}} \quad \frac{70}{56}$$

$$\frac{\dfrac{\underline{|n}}{\underline{|r \quad 1}\,\underline{|n-r \quad 1}}}{\dfrac{\underline{|n}}{\underline{|r \quad 2}\,\underline{|n-r \quad 2}}} = \frac{5}{4}$$

$$\frac{\underline{|r \quad 2}\,\underline{|n \quad r \quad 2}}{\underline{|r \quad 1}\,\underline{|n \quad r \quad 1}} \quad \frac{5}{4}$$

$$\frac{(r \quad 2)\underline{|r \quad 1}\,\underline{|n \quad r \quad 2}}{\underline{|r \quad 1}\,(n \quad r \quad 1)\,\underline{|n \quad r \quad 2}} \quad \frac{5}{4}$$

$$\frac{(r \quad 2)}{n \quad r \quad 1} \quad \frac{5}{4}$$

$$4(r \quad 2) \quad 5(n \quad r \quad 1)$$

$$4r \quad 8 \quad 5n \quad 5r \quad 5$$

9r = 5n – 13 --------- (v)

from (iv) and (v)

$4n - 5 = 5n - 13$

$n = 8$

from (iv)

$9r = 4n - 5$

$9r = 4 \times 8 - 5$

$9r = 27$

$r = 3$

i.e. $(r + 1)$th term = $(3 + 1)$th term = 4th term

$(r + 2)$th term = $(3 + 2)$th term = 5th term

$(r + 3)$th term = $(3 + 3)$th term= 6th term

Ques : Evaluate the following figures with Binomial expansion.

(a) $(99)^4$ (b) $(1.1)^5$

Soln. : (a) $(99)^4$ $= (100 - 1)^4 = \{100 + (-1)\}^4$

$$^4c_0(100)^{4-0}\ -1^{\ 0}\quad ^4c_1\ 100^{\ (4\ 1)}(-1)^1$$

$$^4c_2(100)^{4-2}(-1)^2\quad ^4c_3(100)^{4-3}(-1)^3$$

$$^4c_4(100)^{4-4}(-1)^4$$

$$(100)^4\quad 4(100)^3(-1)\quad \frac{\lfloor 4}{\lfloor 2\ \ \lfloor 4-2}\quad (100)^2\quad 1$$

$$\frac{\lfloor 4}{\lfloor 3\ \lfloor 4-3}(100)(-1)^3\quad 1$$

= 96059601 Ans.

(b) $(1.1)^5$ $\quad (1 \quad 0.1)^5 \quad 1 \quad \frac{1}{10}^{5}$

$$^5c_0(1)^{5-0} \quad \frac{1}{10}^{0} \quad ^5c_1(1)^{5-1} \quad \frac{1}{10}^{1} \quad ^5c_2(1)^{5-2} \quad \frac{1}{10}^{2}$$

$$^5c_3(1)^{5-3} \quad \frac{1}{10}^{3} \quad ^5c_4(1)^{5-4} \quad \frac{1}{10}^{4}$$

$$^5c_5(1)^{5-5} \quad \frac{1}{10}^{5}$$

$$1 \quad 5 \quad \frac{1}{10} \quad \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \quad \frac{1}{10}^{2} \quad \frac{\lfloor 5}{\lfloor 3 \lfloor 2} \quad \frac{1}{10}^{3}$$

$$5 \quad \frac{1}{10}^{4} \quad \frac{1}{10}^{5}$$

$$1 \quad 0.5 \quad \frac{10}{100} \quad \frac{10}{1000} \quad \frac{5}{(10)^4} \quad \frac{1}{(10)^5}$$

$$1 \quad 0.5 \quad 0.1 \quad 0.01 \quad 0.0005 \quad 0.00001$$

= 1.61051 Ans.

Ques. The sum of Binomial Coefficients in the expansion $(1 \quad x)^n$ is $(2)^n$. Prove .

Soln. : $\quad (1 \quad x)^n \quad ^nc_0(1)^{n \ 0}(x)^0 \quad ^nc_1(1)^{n \ 1}x^1$

$$^nc_2(1)^{n-2}(x)^2 \quad ..... \quad ^nc_n(x)^n$$ ---------------- (i)

Putting x = 1 in both sides of Eqn (i)

$$(1 \quad 1)^n \quad {}^n c_0 \quad {}^n c_1 \quad {}^n c_2 \quad ..... \quad {}^n c_n$$

$$(2)^n \quad {}^n c_0 \quad {}^n c_1 \quad {}^n c_2 \quad ...... \quad {}^n c_n$$

Ques. The sum of the coefficients of even term is equal to the sum of the coefficient of odd term in the expansion of $(1 \quad x)^n$.

Soln. : $(1 \quad x)^n \quad {}^n c_0 (1)^{n-0} x^0 \quad {}^n c_1 (1)^{n-1} x^1$

$${}^n c_2 (1)^{n \; 2} x^2 \quad {}^n c_3 (1)^{n \; 3} x^3 \quad .... \quad {}^n c_n (x)^n \text{ ------- (i)}$$

Putting n = –1 in both sides we get

$$(1-1)^n \quad {}^n c_0 - {}^n c_1 \quad {}^n c_2 - {}^n c_3 \quad {}^n c_4 \quad .....$$

$$0 \quad {}^n c_0 - {}^n c_1 \quad {}^n c_2 - {}^n c_3 \quad {}^n c_4 ....$$

$${}^n c_1 \quad {}^n c_3 \quad {}^n c_5 \quad .... \quad {}^n c_0 \quad {}^n c_2 \quad {}^n c_4$$

Ques. If n is even, show that ${}^n c_1 \quad {}^n c_3 \quad {}^n c_5 \quad .... \quad {}^n c_0 \quad {}^n c_2 \quad {}^n c_4 \quad (2)^{n-1}$

Soln. : $(1 \quad x)^n \quad {}^n c_0 (1)^{n-0} x^0 \quad {}^n c_1 (1)^{n \; 1} x^1$

$${}^n c_2 (1)^{n-2} x^2 \quad {}^n c_3 (1)^{n \; 3} x^3 \quad ... \quad {}^n c_n x^n \textbf{ -------- (i)}$$

Putting x = 1 in Eqn (i) we get

$$(1 \quad 1)^n \quad {}^n c_0 \quad {}^n c_1 \quad {}^n c_2 \quad {}^n c_3 \quad {}^n c_n$$

$$(2)^n \quad {}^n c_0 \quad {}^n c_1 \quad {}^n c_2 \quad {}^n c_3 \quad {}^n c_n \text{ ------- (2)}$$

Putting x = –1 in Eqn (i) we get

$$(1-1)^n \quad {}^n c_0 - {}^n c_1 \quad {}^n c_2 \quad {}^n c_3 \quad .....$$

$$0 \quad {}^{n}c_0 -{}^{n}c_1 \quad {}^{n}c_2 -{}^{n}c_3 \quad ..... (3)$$

Adding Eqn (2) and (3) we get

$$(2)^n \quad 2({}^{n}c_0 \quad {}^{n}c_2 \quad {}^{n}c_4 \quad {}^{n}c_6 \quad ...)$$

$$\frac{2^n}{2} \quad {}^{n}c_0 \quad {}^{n}c_2 \quad {}^{n}c_4 \quad {}^{n}c_6 \quad ....$$

$$(2)^{n-1} \quad {}^{n}c_0 \quad {}^{n}c_2 \quad {}^{n}c_4 ...$$

from (3) we know that

$${}^{n}c_1 \quad {}^{n}c_3 \quad {}^{n}c_5 \quad {}^{n}c_0 \quad {}^{n}c_2 \quad {}^{n}c_4$$

Thus ${}^{n}c_1 \quad {}^{n}c_3 \quad {}^{n}c_5 \quad {}^{n}c_0 \quad {}^{n}c_2 \quad {}^{n}c_4 \quad ..... \quad (2)^{n-1}$

Probability

5.5.0 Probability originated in problems dealing with games of chance played by tossing a fair coin, throwing an unbiased die and drawing a card from well shuffled pack. The probability theory deals with uncertain si tuation regarding the occurrence of given phenomena. i.e it is the study of events which are neither absolutely certain nor Impossible.

Classical or Mathematical definition of Probability :

If there are n mutually exclusive, exhaustive and equally likely cases and m of them are favourable to a certain event A, then the probability or chance of the occurrence of A is defined as the

$$P(A)=\frac{\text{Number of favourable cases}}{\text{Total Number of all eqaually likely cases}} \quad \frac{m}{n}$$

Where P(A) denotes the probability of occurrence of the event A

The probability or chance) of non-occurrence of the event A

$$=\frac{\text{Number of unfavourable cases}}{\text{Total no. of all equally likely cases}}$$

Symbolically if A\` or A^c denotes the non-occurrence of A then

$$P(A')=\frac{n-m}{n} \quad \frac{n}{n} \quad \frac{m}{n}$$

$$P(A')=1-\frac{m}{n}, \qquad P(A')=1-P(A)$$

$$\boxed{P(A)+P(A')=1}$$

(a) The odds in favour of the event A

$$=\frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$$

(b) The odds against the event A

$$=\frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$$

The value of probability lies between 0 to 1 when the occurrence of the event is an absolute certain then the value of probability is 1, when the occurrence of the event is an absolute impossible the value of probability is zero.

Ques. : A bag contains three white and five black balls. What is the chance that a ball drawn at random will be black ? Also find the odds in favour of the event and against the even.

Soln. : The total number of balls = 3 + 5 = 8

1 ball can be drawn out of these 8 balls in $8c_1$ ways = 8 ways.

The total number possible cases for the event = 8

Again one black ball can be drawn out of 5 black balls in ${}^5c_1$ ways

$$=\frac{|\underline{5}}{|\underline{1}\,|\underline{5-1}} \quad \frac{|\underline{5}}{|\underline{1}\,|\underline{4}} = 5 \text{ ways.}$$

Thus the chance that the ball drawn is black

$$= \frac{\text{Number of favourable cases}}{\text{Total no of all equally likely cases}}$$

$$P(A) = \frac{5}{8}$$

Total number of cases unfavourable to the even = 8 – 5 = 3

(a) The odds in favour of the event (A)

$$= \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}} = \frac{5}{3}$$

(b) The odds in Against event A

$$= \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}} = \frac{3}{5}$$

Ques; if one card is drawn at random from a well shuffled pack of 52 cards. Find the chance that the card is (i) a diamond (ii) not a diamond (iii) An Ace (iv) neither a spade nor a heart.

Soln. : One card can be drawn out of 52 in ${}^{52}c_1$

$$\text{ways} = \frac{|\underline{52}}{|\underline{1}\,|\underline{52-1}} \quad \frac{|\underline{52}}{|\underline{1}\,|\underline{51}} \quad 52$$

Thus total number of all possible equally likely cases = 52

(i) One diamond can be drawn out of 13 in ${}^{13}c_1$

$$\text{ways} = \frac{|\underline{13}}{|\underline{1}\,|\underline{13-1}} \quad \frac{|\underline{13}}{|\underline{1}\,|\underline{12}} \quad 13 \text{ ways.}$$

Hence, the required chance that the card is a diamond

$$\frac{13}{52} \quad \frac{1}{4} \quad P(A)$$

(ii) The Required chance that the card is not a diamond =P(A´) = 1 – P(A)

$$1-\frac{1}{4} \quad \frac{3}{4}$$

(iii) An Ace can be drawn out of 4 in ${}^4c_1\ ways$ = 4 ways.

No. of favourable cases = 4

Hence, required chance that the card is an a cec $\frac{4}{52} \quad \frac{1}{13}$

(iv) There are 13 spades and 13 heads in a pack 52 cards.

Either a spade or a heart can be drawn in $26c_1$ ways = 26 ways.

The chance that the card is either a spade or a heart $\frac{26}{52} \quad \frac{1}{2}$

Hence the required chance that the card is neither a spade nor a heart

$$1-\frac{1}{2} \quad \frac{1}{2}$$

Ques. A man drawn at random 3 balls from a bag containing 6 red and 5 white balls. What is the chance of getting the ball all red ?

Soln. Total no. of balls = 6 + 5 = 11

3 balls can be drawn out of 11 in $11c_3$ ways

$$\frac{\lfloor 11}{\lfloor 3 \lfloor 8} \quad \frac{11 \quad 10 \quad 9 \quad \lfloor 8}{3 \quad 2 \quad 1 \quad \lfloor 8} \quad 165\,\text{ways}\ .]$$

Thus total number of possible cases for the event = 165.

Again 3 red balls can be drawn out of 6 in ${}^6c_3$ ways

$$\frac{\lfloor 6}{\lfloor 3 \lfloor 3} \quad \frac{6 \quad 5 \quad 4 \quad \lfloor 3}{\lfloor 3 \quad 3 \quad 2 \quad 1} \quad 20$$

Hence the required chance of getting the ball all red

$$\frac{20}{165} \quad \frac{4}{33}$$

Three balls are drawn at random from a bag containing 6 blue and 4 red balls What is the chance that two balls are blue are one ball is red ?

Soln. : Total number of balls = 6 + 4 =10

3 balls can be drawn out of 10 in $^{10}c_3$ ways

$$\frac{|\underline{10}}{|\underline{3}\,|\underline{10-3}} \quad \frac{|\underline{10}}{|\underline{3}\,|\underline{7}} \quad \frac{10 \quad 9 \quad 8 \quad |\underline{7}}{|\underline{7} \quad 3 \quad 2 \quad 1} \quad 120$$

Thus total no. of cases for the event = 120

Again 2 blue balls can be drawn out of 6 in $6c_2$ ways and 1 red ball can be drawn out of 4 in $^4c_1$ way.

Thus 2 blue balls and 1 red balls jointly can be drawn in $^6c_2 \quad ^4c_1\, ways \quad 60$ ways

Hence the required chance of drawing 2 blue and 1 red balls

$$\frac{60}{120} \quad \frac{1}{2}$$

Ques. : What is the chance that a leap year selected at random will contain

(i) 53 sundays (ii) 53 thursday or 53 Fridays.

Soln. : A leap year consists of 366 days in which there are 52 complete weeks and 2 more consecutive days. 52 weeks contains 52 sundays, 52 Thursdays and 52 Fridays.

(i) A leap year will consists 53 Sunday if these two consecutive days contain one Sunday

2 Consecutive days can be selected from the 7 days in a week in the following possible ways.

(Sunday, Monday), (Mon, Tues), (Tues, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat), (Sat, Sunday).

We see that but of the above 7 possibility 2 (Contains) Sunday i.e. No. of favourable cases = 2

Hence he required chance that a leap year will contain 53 Sunday $\frac{2}{7}$

(ii) Again we see that out of the 7 out-comes (ie. Possibilities) 3 are favourable to the event of 53 Thursdays or 53 Friddays and they are (Wednesday, Thurs), Thurs, Fri) and (Fri, Sat).

A/c to Question either thursday or Friday We mean, that either, thursday or Friday or both thursday and Friday)

Hence the required chance that a leap year will contain either 53 thursday or 53 Friday $\frac{3}{7}$ Ans.