B.C.A .

PART - II

SUBJECT
STATISTICS AND LINEAR PROGRAMME TECHNIQUE

PAPER - XV

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ADVANTAGES OF LINEAR PROGRAMMING

* Linear programming helps in making the optimum use of productive resources.

* The approach of decision making using this technique is more practical, systematic and logical.

* The Linear programming technique provides possible solutions since there are other consraints as well that are operating outside the problem which must be taken into account.

  Necessary modification of its mathematical solution is required because all the units produced though are in large quantity, they all can't be sold.

* The most important advantage of this technique is high lighting of bottle necks in the production process.

  This is because if a bottle neck occurs, some machines may not be able to meet demand or may remain idle for some amount of time.

* Risk of tampering the real object is reduced, when a model of the real system is subjected to experimental analysis

* The model will avoid the duplication work in solving the problem.

* Model provide distilled economic descriptions and explanations of the operation of the the system they represent.

LIMITATIONS OF LINEAR PROGRAMMING

* Linear programming treats all the relationships as linear but in real life neither the objective functions nor the constraints are linearly related.

* There is no guarantee that we will get integer valued solution and rounding off to the nearest will not yield an optimal solution in such cases, integer programming is used.

* The linear proramming model does not considers that effect of time and uncertainity.

* Parameters appearing in the model are assumed to be constant but this is not the case in real life.

* It deals with single objective but in real life, the decision maker may come across many multi objective problems. (In such cases a goal programming model is used)

* Models are constructed only to understand the problem and attempt to solve the problem; they are not considered as real problem or system.

* The validity of any model can be conducting the experimental analysis and with relevant data characeristics.

GENERAL MATHEMATICAL MODEL IN LINEAR PROGRAMMING (GLPP)

The general Linear Programming Problem (or model) with decision variables and constraints can be stated in the following form.

Let Z be a linear function on relation defind by

$Z = c_1x_1 + c_1x_2 + ... + c_nx_n$

where, c's are constants

Let (a ij) be an n x n (real) material and

Let $\{b_1, b_2, b_3 ..... bn\}$ be a set of constants

such that

$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq, =, \geq b_1$

$a_{21}x1 + a_{22}x_2 + ... + a_{2n}x_n \leq, =, \geq b2$

$a_{n1}x1 + a_{n2}x_2 + ... + a_{nn}x_n \leq, =, \geq b_n$

and

$x_1, x_2, ..., x_n \geq 0$

The problem of determining n-tuples $(x_1, x_2, ...x_n)$ which makes Za maximum (or minimum) and which satisfies (2) & (3) is called the general linear programming problem.

The above formulation can also be expressed in acompact form using summation sign optimize (maximize or minimize) Z i.e.

optimize (Max. or Min.) $\sum_{j=1}^{n} c_j x_j$ ------- (1)

subject to the linear constraints

$\sum_{j=1}^{n} a_{ij}x_j (\;\;, = \;\;) bi, i = 1, 2$ ----------- (2)

And

$x_j \geq 0, j = 1, 2, ...n$ (3)

PROPERTIES OF LINEAR PROGRAMMING MODEL
* The relationship between variables and constraints must be linear
* The model must have an objective function
* The model must have structural constraints
* The model must have non-negativity constraint.

DEFINITIONS

* Objective functions

  The linear function which is to be maximized (or minimized) in the form :

  $Z = C_1x_1 + C_2x_2 + ... + C_nx_n$

* Constraints

  The inequations are called constraints

  i.e. $a_{M1}x_1 + a_{M2}x_2 + ... + a_{mn}x_n \leq, = \geq b_m$

* Non-negative restriction

  The set of inequations which represent the limit or range of decision variable (non-negative conditions)

  i.e. $x_i \geq 0; i = 1, 2, 3, ... n$

* Solution

  An n-tuple ($x_1, x_2, ..., x_n$) of real numbers which satisfies the constraints is called a solution to the general LP problem

* Optimum solution

  Any feasible solution which optimizes the objective function.

* Unbounded solution

  A solution that can indefinitely increase or decrease the value of objective function of the LP problem.

SOLUTIONS

The set of values of decision variables xj (j = 1,2, ..., n) that satisfies the constraints of an LP problem is said to constitute the solution to that LP prpblem.

BASIC SOLUTION :

* Basic solution

  For a set of m simultaneous equations in n-variables (n > m), a solution obtained by setting (n - m) variables equal to zero and solving for remaining m equations in m variables is called a basic solution.

  The m variables which may be all different from zero are called basic variables.

  The (n-m) variables whose value did not appear in the solution are called non-basic variables.

  The basic solutions are of two types.

* Non-degenerate : if none of the basic variables is zero, the solution is non-degenerate. Basic solution.

* Degenerate : if one or more of the basic variables vanish the solution is called degenerate basic solution.

FEASIBLE AND BASIC FEASIBLE SOLUTION

* Feasible Solution

  The set of values of the decision variables x_j(j = 1, 2, ...n) that satisfy all the constraints and non-negativity conditions of an LP probalem simultaneously is said to be the feasible solution.

* **Basic Feasible Solution**

  A feasible solution to an LP problem which is also the basic solution is called the basic feasible solution i.e. Basic feasible solution is a basic solution which also satisfies the non-negativity conditions. Thus all the basic variables are non-negative values. The are of two types :

* Non degenerate : The basic feasible solution which has all the M basic variables as positive values.

  (the remaining n-M variables are zero)

* degenerate : if atleast one basic variable is zero.

* **SOME IMPORTANT TERMS :**

(i) Infeasible solution

The set of values of decision variables xj (j = 1, 2, ... n) that do not satisfy all the constraints and non-negativity condition of an LP problem simultaneously.

(ii) Optimum Basic feasible solution (optimum solution) :

A basic feasible solution that optimizes the objective function value of the LP problem.

SUMMARY

This unit mainly presented basic assumptions, limitations, components of any linear programming model and broad application areas of linear programming. The major guideling of mathematical modelling of any decision problem are explained using various examples. The following exercises will help the students in dealing with more complex real life problems.

QUESTIONS OF EXERCISES

Problems 1.1

(a) What is Linear Programming ? What are its major assumptions and limitations.

(b) Give some real life applications of Linear Programming

Problem 1.2

Describe the role of linear programming in managerial decision making, bringing out limitations, if any.

Problem 1.3

A firm can produce two types of cloth, say A and B. Three kinds of wool are required for it : red wool, green wool and blue wool. Further, one unit length of type A cloth requires 2 meters of red wool and 3 meters of blue wool. One unit length of type B cloth requires 3 meters of red wool, 2 meters of green wool and 5 meters of blue wool. The firm makes a profit of Rs. 3.00 by selling one metre of cloth of type A and Rs. 5.00 by selling one metre of cloth of type B. Determine how many meters of each type of cloth should be prepared so as to earn the maximum profit. Write down the problem in mathematical form.

(**Hint :** $Z = 3x_1 + 5x_2$

constraints :

$2x_1 + 3x_2 \leq 12$

$2x_2 \leq 10$

$3x_1 + 5x_2 \leq 15$

$x_1 \geq 0, x_2 \geq 0$)

Problem 1.4 :

Suppose that 8, 12 and 9 units of proteins, carbohydrates and fats respectively are the minimum weekly requirements for a person. Food A contains 2, 6 and 1 units of proteins, carbohydrates and fats respectivley per kilogram. If A costs Rs. 8.00 per kilogram and B costs Rs. 4.00 per kilogram. How many kilograms of each one should by per week to minimize his cost and still meet his minimum requirements. Construct a mathematical model.

Problem 1.5 :

A farmer has a 100 acre farm. He can sell all tomatoes, lettuce or radishes and can get a price of Rs. 1.00 per kg for tomatoes, Rs. 0.75 a heap for littuce and Rs. 2.00 per kg for radishes. The average yield per acre is 2,000 kg of tomatoes, 3000 heaps of lettuce and 1000 kg of radishes. Fertilizers are available at Rs. 0.50 per kg and the amount required per acre is 100 kg each for tomatoes and lettuce and 50 kg for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man days of labour are available at Rs. 20 per man-day. Formulate this problem as a linear programming model to maximize the farmer's total profit.

Problem 1.6 :

Egg contains units of vitamin A and 7 units of vitamin B per grams and costs 12 paise per gram. Milk contains 8 units of vitamin A and 12 units of vitamin B per gram and costs 20 paise per gram. The dialy minimum requirement of vitamin A and vitaminB are 100 units and 120 units respectively. Find the optimal produce min.

Problem 1.7 :

An electric appliance company produces two products : Refrigerators and ranges. Production takes place in two separate departments I and II. Refrigerators are produced in department I and ranges in department II. The company's two procucts are sold on a weekly basis. The weekly production cannot exceed 25 refrigerator and 35 range. The company regulalry employs 60 worker in the two department. A refregerator requres 2 man-weeks labour while a range requres..

One man week labour. A refrigerator contributes a profit of Rs. 60 and a range contributes a profit of Rs. 40. How many units of refrigerators and ranges should the company produce to realize the maximum profit. Construct a mathematical model.

Problem 1.8 :

The standard weight of a special purpose brick is 5 kg and it contains two ingredients B_1 and B_2, B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and a minimum of 2 kg of B_2 since the demand for the procuct is likely to be related to the price of the brick. Formulate the above problem as a LP model.

Problem 1.9 :

A company produces two ypes of hat. Each hat of the first type requires twice as much labour time as the second type. If all the hats are of second type only the company can produce a total of 500 hats a day. The market limits daily sales of the first and secon types to 150 and 250 hats. Assumming that the profits per hat are Rs. 8 for type A and Rs. 15 for type B, formulate the problem as a linear programming model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Problem 1.10 :

A company manufactures 3 products P, Q and R. The profits are Rs. 3, Rs. 2 and Rs 4 respectively. The company has two mechines and given below is the required processing time in minutes for each machine on each product.

Machine	Products P	Q	R
I	4	3	5
II	2	2	4

Solutions and Hints :

Ans 1.4 Min $Z = 85x + 40y$

Subject to $2x + y \geq 8$, $6x + y \geq 12$, $x + 3y \geq 9$; $x, y \geq 0$.

Ans 1.5 Max $Z = 1850\, x_1 + 2080x_2 + 1875x_3$

Subjet to $x_1 + x_2 + x_3 \leq 100$

$5x_4 + 6x_2 + 5x_3 \leq 400$

$x_1, x_2, x_3 \geq 0$

Ans. 1.6 Min $Z = 12x_1 + 20x_2$

Subject to $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$x_1, x_2 \geq 0$

Where x_1 and x_2 are the number of units of egg and milk respectvely.

Ans. 1.8 Min $Z = 5x_1 + 8x_2$

Subject to $x_1 \leq 4$

$x_2 \leq 2$

$x_1 + x_2 = 5$ (strength constraints)

$x_1, x_2 \geq 0$, x_1 and x_2 are the arrangments of B_1 and B_2 in kg in the bricks

Ans. 1.9 Max $Z = 8x_1 + 5x_2$

Subject $2x_1 + x_2 \leq 500$

$x_1 \leq 150$

$x_2 \leq 250$

$x_1, x_2 \geq 0$

Where x_1 and x_2 are the number of units of hats of type A and type B respectively.

SUGGESTIVE BOOK READING

* Hamdy A Taha, 1999, Introduction to operations Reasearch, PHI Limited, New Delhi
* Sharma J.K., 1989, Mathematical models in Operations Research, Tata McGraw Hill Publishing
* Beer, Stafford, 1966 Decision and control, John wiley and Sons. Inc., New York