Nalanda Open University

B.SC Part-3

Course : Physics (Hons)

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Topic- Condensed matter (Hall Effect)

The Hall Effect

In 1879, E. H. Hall observed that when a current-carrying conductor is placed in a transverse magnetic field, the Lorentz force on the moving charges produces a potential difference perpendicular to both the magnetic field and the electric current. This effect is known as the Hall effect. Measurements of the Hall voltage are used to determine the density and sign of charge carriers in a material, as well as a method for determining magnetic fields.

In a conductor, the flow of electric current is the movement of charges due to the presence of an electric field. If a magnetic field is applied in a direction perpendicular to the direction of motion of the charges, the moving charges accumulate such that opposite charges lie on opposite faces of the conductor. This distribution of charges produce a potential difference across the material, that opposes the migration of further charge. This creates a steady electrical potential as long as the charges are flowing in the material and the magnetic field is on. This is the Hall effect.

Consider for example, a thin flat uniform ribbon of conducting material, which is oriented so that its flat side is perpendicular to a uniform magnetic field as seen in Figure 1. The electrons are the charge carriers in this model. Assuming the conductor is arranged with length in the x-direction, width w in the y-direction and thickness t in the z-direction as seen below:

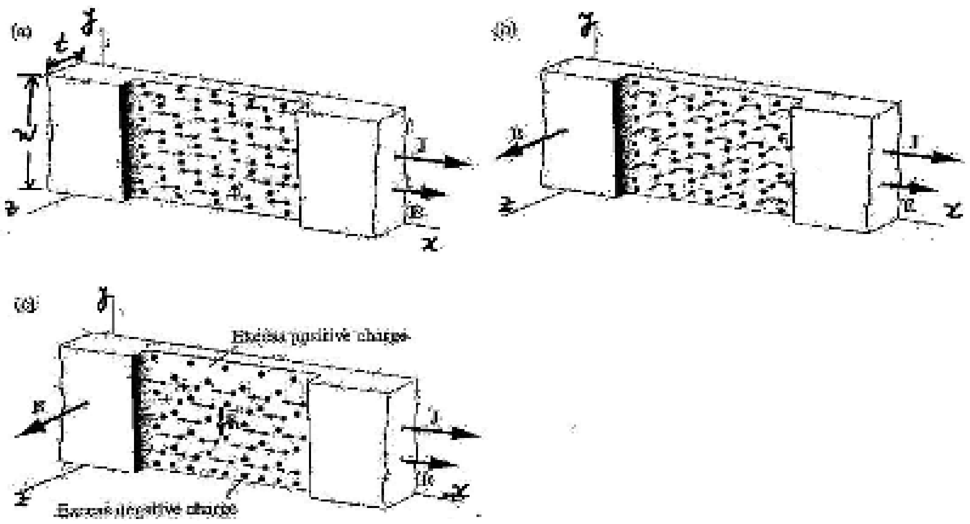

Figure 1: Geometry of the Hall probe

The microscopic form of Ohm's Law relates the current density $J = \frac{I}{A}$ defined as current I per cross-sectional area A of conductor and the microscopic properies of the conducting material: drift velocity v_d, number of charge carriers per unit volume n and the average time τ:

$$\vec{J} = -ne\vec{v_d} = ne^2\tau\frac{\vec{E}}{m} \tag{1}$$

where e is the electronic charge: $e = 1.6 \times 10^{-19}$, and m is the mass of the electron. The drift velocity v_d is the average velocity of the charge carriers over the volume of the conductor. Each charge carrier moves in a random way, undergoing collisions with the lattice. The average time between collisions is τ. Only under the influence of an applied electric field $\vec{E}$ there will be a net transport of carriers along the conductor. Ohm's Law in microscopic form also relates the current density $\vec{J}$ with the electric field applied $\vec{E}$:

$$\vec{J} = \sigma\vec{E} \tag{2}$$

where σ is the electric conductivity.
Using Newton's second law, we can also write:

$$m\vec{a} = \vec{F} = -e\vec{E} \tag{3}$$

and:

$$\vec{a} = \frac{\vec{v_d}}{\tau} \tag{4}$$

In Equation (3), $\vec{a}$ is the average acceleration over a time τ.

Using Eqs. (1)-(3), we obtain:

$$J = \frac{I}{A} = \frac{Q}{At} = \frac{nALe}{A\frac{L}{v_d}} = nev_d \tag{5}$$

When placing the current-carrying conductor in a magnetic field B, the total force has to include the Lorentz force:

$$\frac{m\vec{v_d}}{\tau} = -e(\vec{E} + \vec{v} \times \vec{B}) \tag{6}$$

Assuming the magnetic field in the z-direction, as in Fig.1 and defining the cyclotron frequency $\omega_c = \frac{eB}{m}$, we can write the three components of the drift velocity:

$$v_{d_x} = -\frac{e\tau}{m}E_x - \omega_c \tau v_{d_y} \tag{7}$$

$$v_{d_y} = -\frac{e\tau}{m}E_y + \omega_c \tau v_{d_x} \tag{8}$$

$$v_{d_z} = -\frac{e\tau}{m}E_z \tag{9}$$

With current density J in the x-direction, the magnetic field will drift electrons from the conductor along the negative y-direction, leading to a charge buildup and therefore to an electric field E_y within the conductor. When steady state is reached, the drift down along the negative y-direction stops, electric force due to E_y will cancel the action of magnetic force and the current density will be strictly along the x-direction. Using Equation (7) and steady-state condition: $v_{d_x} = 0$, we obtain the Hall field:

$$E_y = v_{d_x} B_z \tag{10}$$

The Hall voltage is the potential difference across the sample: the quantity you'll measure in this experiment. It is related to the Hall field by:

$$V_H = -\int_0^w E_y dy = -E_y w \tag{11}$$

From equations (5), (7) and (8), we obtain:

$$E_y = -\frac{eB\tau}{m}E_x \tag{12}$$

A convenient experimental quantity is the Hall coefficient, R_H defined as:

$$R_H = \frac{E_y}{J_x B} = \frac{eB\tau E_x}{m}\frac{m}{ne^2\tau E_x B} = \frac{1}{ne} \tag{13}$$

The Hall coefficient is positive if the charge carriers are positive and negative if the charge carriers are negative. The SI units of the Hall coefficient are: m^3/C.
Also related to the drift velocity v_d is the *electric mobility* μ, defined as:

$$\mu = \frac{v_d}{E} \tag{14}$$

Using Eq.(5) and Ohm's Law in microscopic form (2), we obtain a very useful relation between the Hall coefficient R_H and the electric mobility μ:

$$\mu = \sigma R_H \tag{15}$$

The quantity measured in this experiment is not conductivity σ but resistivity $\rho = \frac{1}{\sigma}$.

The Hall effect in metals and semiconductors

In order to understand some of the ideas involved in theory of the Hall effect in real materials, it is instructive to construct a more careful model for electric currents under electric and magnetic fields from a classical point of view. We imagine that the charge carriers move in a medium that offers some resistance. The resistance is due to scattering between the carriers and impurities in the material and between the carriers and vibrations of the material's atoms. Each charge carrier is accelerated by the applied fields but every so often it scatters and loses energy. If we assume that the average time between scattering events is τ, then we have, on average, a retarding force acting on the carriers of

$$\vec{F}_{\text{retard}} = -\frac{m\vec{v}}{\tau} , \tag{16}$$

where m is the mass of the carrier. So under the influence of applied electric and magnetic fields, Newton's second law reads

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) - \frac{m\vec{v}}{\tau} , \tag{17}$$

where the velocity $\vec{v}$ is taken to be an average over all of the carriers.

At steady state, the time derivative of $\vec{v}$ will vanish. Under the usual convention that $\vec{B}$ points along the z axis, we obtain the component equations for $\vec{v}$ by setting the left hand side of Eq. (17) to zero and rearranging:

$$\begin{aligned} v_x &= \frac{q\tau}{m} E_x + \frac{q\tau}{m} B_z v_y \ , \\ v_y &= \frac{q\tau}{m} E_y - \frac{q\tau}{m} B_z v_x \ , \\ v_z &= \frac{q\tau}{m} E_z \ . \end{aligned} \tag{18}$$

From Eq. (1) we have that $J_x = nqv_x$ (and correspondingly for y and z components). By solving the above equations for v_x, v_y and v_z in terms of the components of $\vec{E}$ and B_z we get

$$\begin{aligned} J_x &= \frac{\sigma}{1+(\omega_c\tau)^2} \left(E_x + \omega_c \tau E_y\right) \ , \\ J_y &= \frac{\sigma}{1+(\omega_c\tau)^2} \left(E_y - \omega_c \tau E_x\right) \ , \\ J_z &= \sigma E_z \ , \end{aligned} \tag{19}$$

where

$$\sigma \equiv \frac{nq^2\tau}{m} \ , \tag{20}$$

and

$$\omega_c \equiv \frac{qB_z}{m} \ . \tag{21}$$

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In the expression for the conductivity σ it may seem that all we have done is to replace one unknown quantity $\vec{v}$ with another unknown quantity τ. But the parameter τ, called the *relaxation time*, is widely used in discussions of electronic transport in materials, and can be justified in a quantum-mechanical context via the Boltzmann transport equation

The angular frequency ω_c is known as as the "cyclotron frequency". It is the frequency of rotation of a charge in a magnetic field, and can be taken as a measure of the strength of the field. The combination $\omega_c\tau$ is used to characterize an experimental situation: if the magnetic field is weak and/or the relaxation time short, $\omega_c\tau \ll 1$ and our experiment is in the "weak-field limit"; alternately if $\omega_c\tau \gg 1$ the experiment is in the "strong-field limit". A number of materials show strikingly different behavior between the weak- and strong-field limits; aluminum is one.

In our classical model of the Hall effect with a single type of charge carrier, however, there is no such crossover between the weak and strong field.