B.C.A -PART - I

SUBJECT : MATHEMATICS

PAPER-IV

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UNIT 3 MATRICES

3.1 Introduction :

The theory of matrices was originated in its necessity of solving a system of linear simultenous equations. Prof. A cayley made a valuable contribution in matrix algebra. Now a days, matrix algebra occupies a central place in different branches of science and technology, specially in computer science, operations Research, statistics etc.

We consider the following system of linear equations.

$$\left.\begin{aligned} 2x - 3y + z &= 1 \\ x + 2y - 4z &= -1 \\ 3x - 4y - z &= 4 \end{aligned}\right\}$$

If we arrange the coefficients of x, y and z in the order in which they occur, in the given equation and enclose them by big brackets, we shall obtain the following array of numbers, written as

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 3 & -4 & -1 \end{bmatrix}$$

The above representation of numbers will be termed as a matrix

3.1.1 Definition of a matrix :

A matrix is a rectangular array of numbers written in terms of (horizontal) rows and (vertical) columns, written under two big brackets, but there is no value assigned to it.

We denote a matrix by capital letters of english alphabet such as A, B, C, X etc.

Ex. (i) Let $A = \begin{bmatrix} 2 & -1 & 4 & 1 \\ 3 & 1 & 2 & 4 \end{bmatrix}$

This is a matrix having 2 rows and 4 columns. We call this matrix A to be of order (2×4)

Ex. (ii) Let $B = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 3 & 1 & 2 \\ 1 & -2 & 1 \end{bmatrix}$

This is a matrix having 4 rows and 3 columns. We call this matrix to be of order (4×3)

Repretentation of a matrix

A matrix having 3 rows and 4 columns can be represented as

$$A\begin{bmatrix} a_{11} & a_{12} & a_{43} & a_{44} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

This is a matrix of order (3 × 4)

We use double suffices to represent the element of this matrix where the first suffix denote the position of row and the second suffix the position of column according to the position of the element in this matrix.

Ex. a_{13} = element occuring in first row and third column

a_{12} = element occuring in third row and second column

In general we write

a_{ij} = (i, j) the element

= element occuring in ith row and jth column

Similarly, a matrix having m rows and n columns can be written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & ..... & a_{1n} \\ a_{21} & a_{22} & a_{23} & .... & a_{2n} \\ .... & .... & .... & ..... & .... \\ .... & .... & .... & .... & .... \\ a_{ml} & a_{m2} & a_{m3} & ..... & a_{mn} \end{bmatrix}$$

This is a matrix of order (m × n)

It can also be written as

$$A = \left[a_{ij}\right]_{m\times n}, \text{when } |\le i \le m, | \le j \le n, i, j \in N$$

3.2. Different types of matrices :

3.2.1 Zero matrix of Null matrix :

A matrix whose every element is zero, is called a zero matrix.

So $A = \left[a_{ij}\right]$ is a zero matrix, if

Ex (i) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

3.2.2.Square matrix :

A matrix whose number of rows and columns are equal, is called a square matrix

So that matrix $A = \left[a_{ij}\right]_{m \times n}$ is a square matrix if m = n

Ex. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \left[a_{ij}\right]_{3 \times 3}$

Then A is a square matrix of order (3 × 3)

This matrix A is also a square matrix of order 3.

Determinant of a square matrix

|A| = determinant of $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Ex. If $A = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{vmatrix}$ Expanding by first row

$= 4\ (18 - 24) - 5(9 - 21) + 6(8 - 14) = 0$

Principal diagonal

In the square matrix A, given above, the element a_{11}, a_{22}, a_{33} are called the diagonal elements of A. The line along which these elements a_{11}, a_{22}, a_{23} lie, is called the principal diagonal of the matrix A

Ex. A square matrix of order 3 is

$$A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 4 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

Here the diagonal elements are 2, 4, 3. Also the line along which 2, 4, 3 lie is the principal diagonal of A.

3.2.3 Rectangular matrix :

A matrix $A = \left[a_{ij}\right]_{m \times n}$ is called a rectangular matrix if $m \neq n$

Ex. $\begin{bmatrix} 1 & 3 & 2 \\ 4 & -2 & 5 \end{bmatrix}_{2\times3}$ $(\because 2 \neq 3)$

3.2.4 Row matrix and column matrix :

(i) Row matrix

A matrix having only one row is called a row matrix or row vector of the given matrix A

(ii) Column matrix

A matrix having only one column is called a column matrix or column vector of the given matrix A.

Ex. Let $A = \begin{bmatrix} 1 & -3 & 4 & 2 \\ 2 & 1 & 3 & -1 \\ 4 & -1 & 5 & 3 \end{bmatrix}$

be a matrix of order (3 × 4)

Then $[1,-3, 4, 2]_{1 \times 4}$, $[4, -1, 5, 2]_{1 \times 4}$ are row matrices of A

Also $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}_{1}$, $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}_{3\times1}$ are column matrices of A

3.2.5 Diagonal matrix :

A square matrix A, whose all elements except the principal diagonal elements are zero, is clled a diagonal matrix.

So the square matrix $A = \left[a_{ij}\right]$ is a diagonal matrix, if $a_{ij} = 0$ for all $i \neq j$

Ex. Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Here A is a diagonal matrix of order (3 × 3)

It can also be written as A = diag [2,–3, 5]

3.2.6 Unit Matrix : or Identify Matrix (I)

A square matrix A, whose all elements in the principal diagonal are 1 and other elements are all zero, is called a unit matrix

So the square matrix $A = \left[a_{ij}\right]$ is a unit matrix

if $a_{ij} = 1$, when i = j

= 0, when $i \neq j$

Ex. (i) $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (ii) $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3.2.7 Scalar Matrix :

A square matrix whose elements in the principal diagonal are equal to some constant and other elements are all zero, is called a scalar matrix.

So the square matrix $A = [a_{ij}]$ is a scalar matrix

if $a_{ij} = k$, when i = j

= 0 when $i \neq j$

Ex. (i) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

3.2.8 Equal matrix :

Two matrices A and B are said be equal, written as A = B, if

(i) The matrices A and B are of the same order

(ii) The corresponding elements of A and B are equal

Ex. If $\begin{bmatrix} 2 & x & -1 \\ 4 & -3 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 & z \\ 4 & -3 & 5 \end{bmatrix}$, find x, y and z

Here both the matrices are of the same order (2 × 3)

Also, equality of matrices gives

x = 3, 4 = y , – 1 = z $\therefore$ x = 3, y = 4, z = – 1

3.3. Operation on Matrices :

3.3.1 Addition of two matrices :

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same order (m × n). The addition of A and B. written as A + B, is also a matrix of the same order. (m×n), whose elements are obtained on adding the corresponding elements of the two matrices A and B.

So $A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$

This matrix is also of order (m × n)

Ex. If $A=\begin{bmatrix}2 & -4 & 1\\ 3 & 5 & 2\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4 & 2\\ -1 & 2 & 3\end{bmatrix}$, find A + B

Here A and B are matrices of the same order (2 × 3)

$$\therefore\ A+B=\begin{bmatrix}2 & -4 & 1\\ 3 & 5 & 2\end{bmatrix}+\begin{bmatrix}3 & 4 & 2\\ -1 & 3 & 5\end{bmatrix}, \text{find } A+B$$

$$=\begin{bmatrix}2+3 & -4+4 & 1+2\\ 3+(-1) & 5+3 & 2+5\end{bmatrix}=\begin{bmatrix}5 & 0 & 3\\ 2 & 8 & 7\end{bmatrix}$$

Properties of addition of matrices

1. If A be any matrix of order (m × n) and O be the zero matrix of order (m×n), Then A + O = O + A = A
2. If A and B be two matrices of the same order (m × n) then A + B = B + A (Commutative law of addition)
3. If A, B and C be three matricess of the same order (m × n) Then A + (B + C) = (A + B) + C (Associative law for addition).

3.3.2. Scalar Multiplication of a Matrix :

Let A be any matrix of order (m×n) and K be any non-zero constant, then the scaler multiplication of A by k denoted by K.A is also matrix of order (m×n), whose elements are obtained on multiplying each element of the matrix A by K.

$$\therefore kA = k\left[a_{ij}\right]=\left[k.a_{ij}\right]$$

Ex. If $A=\begin{bmatrix}2 & -3 & 4\\ 3 & 4 & 1\\ -1 & 5 & 3\end{bmatrix}=$ Then find 5A and $2A\begin{bmatrix}5(2) & 5(-3) & 5(4)\\ 5(3) & 5(4) & 5(1)\\ 5(-1) & 5(5) & 5(3)\end{bmatrix}$

$$=\begin{bmatrix}10 & -15 & 20\\ 15 & 20 & 5\\ -5 & 25 & 15\end{bmatrix}$$

Also $-2A=(-2)\begin{bmatrix}2 & -3 & 4\\ 3 & 4 & 1\\ -1 & 5 & 3\end{bmatrix}=\begin{bmatrix}(-2)2 & (-2)(-3) & (-2)4\\ (-2)3 & (-2)4 & (-2)1\\ (-2)(-1) & (-2)5 & (-2)3\end{bmatrix}$

$$=\begin{bmatrix}-4 & 6 & -8\\ -6 & -8 & -2\\ 2 & -10 & -6\end{bmatrix}$$

Negative of a matrix

If $A=\left[a_{ij}\right]$ be any matrix of order (m×n), then negative of A, written as –A, is the matrix obtained on multiplying each element of the matrix A by –1

So – A= (–1) $\left[a_{ij}\right]=\left[-a_{ij}\right]$

A – B = A + (–B)

$$=\left[a_{ij}\right]+\left[-b_{ij}\right]=\left[a_{ij}-b_{ij}\right]$$

Ex. If $A=\begin{bmatrix}1 & 3 & 4\\ 2 & 1 & -5\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix}$, find A –B

Here – B $=\begin{bmatrix}-2 & 1 & -4\\ -3 & -2 & -5\end{bmatrix}$

So A – B = A + (–B) = $\begin{bmatrix}1 & 3 & 4\\ 2 & 1 & -5\end{bmatrix}+\begin{bmatrix}-2 & 1 & -4\\ -3 & -2 & -5\end{bmatrix}$

$$=\begin{bmatrix}1-2 & 3+1 & 4-4\\ 2-3 & 1-2 & -5-5\end{bmatrix}=\begin{bmatrix}-1 & 4 & 0\\ -1 & -1 & -10\end{bmatrix}$$

Properties of – A

If A is a matrix of order (m×n) then

A+(–A) = (–A) + A = O

Where o is the zero matrix of order (m×n)

Here–A is called the additive inverse of A

Properties of Scalar Multiplication

If A and B be any two matrices of the same order (m×n) and K be any non-zero constant, then

(i) K (A + B) = KA + KB

(ii) K (A – B) = KA – KB

3.3.3 Transpose of a matrix :

If $A=\left[a_{ij}\right]$ be any matrix of order (m×n), then the transpose of A, written as A^1 or A^T, is a matrix of order (n×m), whose elements are obtained on interchanging the rows and columns of the matrix A

So A^1 = Transpose of $A=\left[a_{ij}\right]$

This is matrix of order (n×m)

Ex. If $A=\begin{bmatrix}1 & -3 & 4\\ 2 & 4 & 3\end{bmatrix}$, find A^1

$$\therefore A^1=\begin{bmatrix}1 & -3 & 4\\ 2 & 4 & 3\end{bmatrix}=\begin{bmatrix}1 & 2\\ -3 & 4\\ 4 & 3\end{bmatrix}$$

This is a matrix of order (3×2)

Properties of Transpose

1. If A be any matrix of order (m×n) and $k \neq 0$ be a constant, then

   (i) $\left(A^1\right)^1 = A$ (ii) $(KA)^1 = K\left(A^1\right)$

2. If A and B be any two matrices of the same order (m ×n), then

   (i) $(A+B)^1 = A^1 + B^1$ (ii) $(A-B)^1 = A^1 - b^1$

3.3.4 Symmetric matrix :

A square matrix A is said to be symmetric , of $A^1 = A$

Note. A square metrix $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ is symmetric, if

$a_{12} = a_{21}$; $a_{13} = a_{31}, a_{23} = a_{32}$

If $a_{ij} = a_{ij}$ for all $i \neq j$

Ex. Let $A=\begin{bmatrix}2 & 3 & 5\\ 3 & 4 & -9\\ 5 & -9 & 1\end{bmatrix}$, Then $A^1 = A$

So A is a symmetric matrix

3.3.5. Skew symmetric matrix :

A square matrix A is said to be skew-symmetric if $A^1 = -A$

Note A square matrix $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$

is skew-symmetric, if $a_{ij} = -a_{jj}$ for all ij

This gives

$a_{12} = -a_{21}, a_{23} = -a_{32}; a_{13} = -a_{31}$

$a_{11} = -a_{11}; a_{23} = -a_{23}; a_{23} = -a_{33}$

i.e. $a_{ij} = 0$ for all i = j

and $a_{ij} = -a_{ij}$ for all i ≠ j

Ex. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$

Then $A^1 = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix} = (-1)\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$

= – A

So A is a skew-symmetric matrix

3.3.6 Show that every square matrix A can be uniquely expressed as the sum of a symmetric matrix and a skew symmetric matrix.

Also Let A be any square matrix of order n.

Then we can write

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') \qquad ...(1)$$

Take A = B +C

Where $B = \frac{1}{2}(A + A')$ and $C = \frac{1}{2}(A - A^1)$ ...(2)

First, we show that B is a symmetric matrix and C is a skew-symmetric matrix

Now $B^1 = \left\{\frac{1}{2}(A + A^1)\right\}^1 = \frac{1}{2}\{A + A^1\}$

$= \frac{1}{2}\left\{(A + A^1)^1\right\}$, by property of transpose

$= \frac{1}{2}\{A^1 + A\}$

$= \frac{1}{2}\{A + A^1\} = B$

So, B is a symmetric ma trix

Also $C^1 = \left\{\frac{1}{2}\left(A - A^1\right)\right\}^1 = \frac{1}{2}\left\{A - A^1\right\}^1$

$= \frac{1}{2}\left\{A^1 - \left(A^1\right)^1\right\}$, by property of transpose

$\left(\because \left(A^1\right)^1 = A\right)$

$= \frac{1}{2}\left\{A^1 - A\right\}$

$= -\frac{1}{2}\left(A - A^1\right) = -C$

So C is a skew symmetric matrix

Hence from Eq. (1) it follows that A can be expressed as the sum of a symmetric matrix B and a skew symmetric matric C.

For uniqueness of the representation (1)

Let $A = B_1 + C_1$

Where B_1 is a symmetric matrix and C_1 is a skew-symmetric matrix, so that $B_1^1 = B_1$ and $C_1^1 = -C_1$

$\therefore A^1 = B_1 - C_1$ .... (4)

From Eq (3) and Eq (4) : $A + A^1 = \left(B_1 + C_1\right) + \left(B_1 - C_1\right) = 2B_1$

$\therefore \frac{1}{2}\left(A + A^1\right) = B_1$ ie. $B = B_1$ by Eq.(2)

Also $A - A^1 = \left(B_1 + C_1\right) - \left(B_1 - C_1\right) = 2C_1$

$\therefore \frac{1}{2}\left(A - \left(A^1\right)\right) = C_1$ ie $C = C_1$ by Eq (2)

This shows that the representation A = B +C is unique

3.3 Worked out examples :

1. Determine a matrix $[a_{ij}]$ of order (2 × 4), whose elements are $a_{ij} = \frac{1}{2}(i + j)$

Ans. The matrix $A = \left[a_{ij}\right]$ of order (2 × 4) is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}$$

Here

$$a_{11} = \frac{1}{2}(1+1) = 1,\ a_{12} = \frac{1}{2}(1+2) = \frac{3}{2}, a_{22} = \frac{1}{2}(2+2) = 2$$

$$a_{23} = \frac{1}{2}(2+3) = \frac{5}{2}; a_{24} = \frac{1}{2}(2+4) = 3$$

This given $A = \begin{bmatrix} 1 & 3/2 & 2 & 5/2 \\ 3/2 & 2 & 5/2 & 3 \end{bmatrix}$

2. If $A = \begin{bmatrix} 1 & -3 & 2 & 4 \\ 2 & 3 & 4 & 1 \\ 4 & 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 & 5 \\ -1 & 4 & 2 & 5 \\ 2 & 3 & -4 & 1 \end{bmatrix}$

find (i) A+ B (ii)3A– 2B (iii) $A^1 + B^1$

Ans. Here A and B are matrices of the same order (3×4)

For (i)

$$A + B = \begin{bmatrix} 1 & -3 & 2 & 4 \\ 2 & 3 & 4 & 1 \\ 4 & 1 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 & 5 \\ -1 & 4 & 2 & 3 \\ 2 & 3 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 3 & 9 \\ 1 & 7 & 6 & 4 \\ 6 & 4 & -1 & 3 \end{bmatrix}$$

For (ii)

$$3A - 2B = 3\begin{bmatrix} 1 & -3 & 2 & 4 \\ 2 & 3 & 4 & 1 \\ 4 & 1 & 3 & 2 \end{bmatrix} + (-2)\begin{bmatrix} 3 & 2 & 1 & 5 \\ -1 & 4 & 2 & 3 \\ 2 & 3 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -9 & 6 & 12 \\ 6 & 9 & 12 & 3 \\ 12 & 3 & 9 & 6 \end{bmatrix} + \begin{bmatrix} -6 & -4 & -2 & -10 \\ 2 & -8 & -4 & -6 \\ -4 & -6 & 8 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -13 & 4 & 2 \\ 8 & 1 & 8 & -3 \\ 8 & -3 & 17 & 4 \end{bmatrix}$$

For (iii)

$$A^1 + B^1 = \begin{bmatrix} 1 & -3 & 2 & 4 \\ 2 & 3 & 4 & 1 \\ 4 & 1 & 3 & 2 \end{bmatrix}^1 + \begin{bmatrix} 3 & 2 & 1 & 5 \\ -1 & 4 & 2 & 3 \\ 2 & 3 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ -3 & 3 & 1 \\ 2 & 4 & 3 \\ 4 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 2 & 4 & 3 \\ 1 & 2 & -4 \\ 5 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & 6 \\ -1 & 7 & 4 \\ 3 & 6 & -1 \\ 9 & 4 & 3 \end{bmatrix}$$

3. If $X + Y = \begin{bmatrix} 2 & 4 & -1 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 4 & 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 4 & 2 & -3 & 1 \\ -3 & 6 & 3 & 0 \\ 2 & 3 & 4 & -3 \end{bmatrix}$

Find X and Y

Ans.

$$\therefore (X+Y)+(X-Y) = \begin{bmatrix} 2 & 4 & -1 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 4 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 2 & -3 & 1 \\ -3 & 6 & 3 & 0 \\ 2 & 3 & 4 & -3 \end{bmatrix}$$

ie. $2X = \begin{bmatrix} 6 & 6 & -4 & 4 \\ 0 & 8 & 4 & 4 \\ 6 & 4 & 6 & 2 \end{bmatrix}$

$$X = \frac{1}{2}\begin{bmatrix} 6 & 6 & -4 & 4 \\ 0 & 8 & 4 & 4 \\ 6 & 4 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -2 & 2 \\ 0 & 4 & 2 & 2 \\ 3 & 2 & 3 & 1 \end{bmatrix}$$

Also

$$(X+Y)-(X-Y) = \begin{bmatrix} 2 & 4 & -1 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 2 & -3 & 1 \\ -3 & 6 & 3 & 0 \\ 2 & 3 & 4 & -3 \end{bmatrix}$$

$$\therefore\ 2y = \begin{bmatrix} -2 & 2 & 2 & 2 \\ 6 & -4 & -2 & 4 \\ 2 & -2 & -2 & 8 \end{bmatrix}$$

$$\therefore y = \frac{1}{2}\begin{bmatrix} -2 & 2 & 2 & 2 \\ 6 & -4 & -2 & 4 \\ 2 & -2 & -2 & 8 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 & 1 \\ 3 & -2 & -1 & 2 \\ 1 & -1 & -1 & 4 \end{bmatrix}$$

4. Express the square matrix A as the sum of a symmetric and a skew-symmetric matrix where.

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$$

Ans.

Here $A^1 = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$

We know that the matrix A can be uniquely expressed as A = B +C

Where $B = \frac{1}{2}\left(A + A^1\right) = a$ symmetric matrix

and $C = \frac{1}{2}\left(A - A^1\right) = a$ skew –symmetric matrix

Now, $B = \frac{1}{2}\left(A + A^1\right) = \frac{1}{2}\left\{\begin{bmatrix} 2 & 4 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & -2 \\ 3 & 2 & 4 \end{bmatrix}\right\}$

$$= \frac{1}{2}\begin{bmatrix} 4 & 7 & 4 \\ 7 & 2 & 0 \\ 4 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 2 & 7/2 & 2 \\ 7/2 & 1 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

Also $C = \frac{1}{2}\left(A - A^1\right) = \frac{1}{2}\left\{\begin{bmatrix} 2 & 4 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & -2 \\ 3 & 2 & 4 \end{bmatrix}\right\}$

$$=\frac{1}{2}\begin{bmatrix}0 & 1 & 2\\ -1 & 0 & 4\\ -2 & -4 & 0\end{bmatrix}=\begin{bmatrix}0 & 1/2 & 1\\ -1/2 & 0 & 2\\ -1 & -2 & 0\end{bmatrix}$$

In this way the matrix A can be expressed as B + C, where B is a symmetric matrix and C is a skew-symmetric matrix as given above

3-4 Exercise I

1. (i) Find the matrix $[a_{ij}]$ of order (2 ×3), whose elements are $a_{ij}=\frac{1}{2}(5i-3j)$

Ans. $\begin{bmatrix}1 & -1/2 & -2\\ 7/2 & 2 & 1/2\end{bmatrix}$

(ii) Find the matrix $[a_{ij}]$ of order (3 ×2), whose elements are $a_{ij}=\frac{1}{2}|i-j|$

Ans. $\begin{bmatrix}0 & 1/2\\ 1/2 & 0\\ 1 & 1/2\end{bmatrix}$

2. If $A=\begin{bmatrix}2 & 4 & 1\\ 3 & 2 & 4\\ 4 & 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2 & 1\\ 4 & 3 & -2\\ 2 & 4 & 3\end{bmatrix}$

Find (i) A + B (ii) 5A – 3B (ii) A^1+B^1

Ans. $\begin{bmatrix}5 & 2 & 2\\ 7 & 5 & 2\\ 6 & 5 & 8\end{bmatrix},\begin{bmatrix}1 & 26 & 2\\ 3 & 1 & 26\\ 14 & -7 & 11\end{bmatrix},\begin{bmatrix}5 & 7 & 6\\ 2 & 5 & 5\\ 2 & 2 & 8\end{bmatrix}$

3. (i) Find a matrix X such that $2A+B+X=0$ where

$A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$

Ans. $\begin{bmatrix}-1 & -2\\ -7 & -13\end{bmatrix}$

(ii) $X+Y\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & 7\end{bmatrix}$ and $X-Y=\begin{bmatrix}5 & 4 & 3\\ 0 & 3 & -1\end{bmatrix}$

Find X and Y

Ans. $\begin{bmatrix} 4 & 3 & 4 \\ -1 & 2 & -4 \end{bmatrix}, \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & -3 \end{bmatrix}$

(iii) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 \\ -1 & 2 & 5 \\ 2 & 0 & 5 \end{bmatrix}$

Find a matrix C such that A + C = B

Ans. $\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

04. Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ 1 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix

Ans. $\begin{bmatrix} 1 & -3/2 & 3 \\ -3/2 & 8 & 9/2 \\ 3 & 9/2 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 9/2 & 2 \\ -9/2 & 0 & -3/2 \\ -2 & 3/2 & 0 \end{bmatrix}$